Labour market frictions, endogenous retirement, and wealth

Emil Kostadinov* Melvyn Coles

March 21, 2017

Abstract

Life-cycle theory has long recognised that the properties of earnings processes affect saving decisions. Individual earning profiles are labour-market outcomes and the main life-cycle stages - a sequence of employment and retirement - correspond to labour-market decisions. This paper demonstrates how such *sequence* emerges as a result of optimal labour-market behaviour of risk-averse workers facing search frictions; provides a tractable description of wealth dynamics emphasising the link between workers' labour-market productivity, wealth, retirement plans and saving decisions; explores the implications of efficient retirement choice for the saving decisions of heterogeneous households and studies their relevance in accounting for the empirical wealth distribution; and demonstrates the effect of costly education on the wealth distribution among individuals with different earning ability.

1 Introduction

Respondents to household surveys identify financing consumption after retirement and insurance against unexpected events as the two most important reasons for saving[1]. While

*I acknowledge and appreciate the financial support from the Economics and Social Research Council (ESRC).

[1]For example, see Cagetti (2003) and Schunk (2009) for descriptive statistics on self-reported saving motives.

theoretical models emphasising precautionary motives have proved successful in replicating broad trends in empirical wealth distributions, direct evidence shows that precautionary wealth is too small a proportion of total wealth[2]. Long-term life-cycle motives should instead be the dominant force behind central tendencies of the wealth distribution.

Canonical versions of the life-cycle model[3] take a household's earning process and retirement age as exogenous. In fact, earnings are determined by individual labour market experiences and a large literature studying the interaction between labour market decisions and life-cycle phenomena has emerged. Moreover, the assumption that workers retire at the same age conceals considerable heterogeneity. In the US, labour-force participation declines steadily after the age of 55 (with discontinuous falls around the Social Security early and full retirement ages) but about 20 percent of 70-75 year-olds (12 percent of 75-80 year-olds) are still participants[4]. Retirement is a decision rather than an exogenous feature of the environment. Burbidge and Robb (1980) proposed a modification of the pure life-cycle model where households choose efficiently when to retire in order to maximise lifetime utility from consumption and leisure. Versions of this model have been used extensively to study optimal retirement age[5]. It should be noted that in this tradition labour markets are not modelled explicitly and agents are assumed to follow a sequence of employment and non-participation (retirement) for exogenous reasons.

It has been documented that certain groups of the population (e.g. the poor) save significantly less than implied by plain life-cycle models [6]. Various explanations have been

[2]Fulford (2015) shows that the median household in the SCF reports to need a little more than a month's income in savings for "emergencies and other unexpected things that may come up". Hurst *et al.* (2010) finds that precautionary savings account for less than 10 percent of total household wealth in the PSID, and attribute previous higher estimates (e.g. Carroll and Samwick (1997)) to pooling together business owners and other household - two groups that otherwise hold the same low ratio of precautionary to total wealth. This is in line with earlier studies such as Lusardi (1998) and Guiso *et al.* (1992).

[3]For example, Hubbard *et al.* (1994b) and Gourinchas and Parker (2002).

[4]These are the figures from the 2012 CPS as reported by Toossi (2013).

[5]Recent studies include Bloom *et al.* (2007, 2014), D'Albis *et al.* (2012) and Kuhn *et al.* (2015) who investigate optimal retirement in the context of demographic changes, increases in longevity or expenditure on healthcare.

[6]For example, in an influential study, Dynan *et al.* (2004), using alternative strategies for isolating permanent earnings, documented that saving rates are increasing with measures of permanent earnings consistently across specifications in three different US surveys (PSID, SCF and CEX). Hubbard *et al.* (1994a)

proposed, including ones rooted in behavioural economics[7]. But when retirement is a decision, a rational household's optimal saving policy should be consistent with its plan about when to retire - a logic somewhat explicit in defined contribution plans where workers choose saving rates. If expected time of retirement varies with expected long-term earnings[8], then so will saving behaviour. For example, the poor might save little because they intend to work until older ages. If leisure is a normal good then its pursuit at older ages might be prohibitively costly for some.

This paper presents a model of a frictional labour market where risk-averse workers enjoy leisure when non-participants and earn a wage when employed but transit from non-participation into employment only through a spell of frictional unemployment. Labour is indivisible. Workers are characterised by time-invariant productivity which maps into a wage rate and differ by initial wealth endowments. When sufficiently asset-poor, workers of any productivity plan to work indefinitely. Efficient retirement plans along the lines of Burbidge and Robb (1980) arise endogenously from more asset-rich employed worker's decisions and optimal labour-market and consumption/saving policies are consistent with the retirement plan. Sufficiently asset-rich workers never work, enjoy leisure and consume out of interest income.

The model combines the main abstraction of the life-cycle hypothesis with endogenous

use PSID to show that a significant fraction of households with low lifetime earnings have pre-retirement wealth accumulation too small to be consistent with the perfect-market version of the life-cycle model. They further show that asset-poor households have inconsistently low saving rates, to the extent that low wealth is an "absorbing state over lengthy periods of time".

[7]Examples of rational explanations include persistent differences in time preference rates or subsistence parameters (Dynan *et al.*, 2004); differences in Social Security replacement rates across high and low earning households (Huggett and Ventura, 2000); consumers deriving utility directly from wealth (Carroll, 2000), among others. For examples of behavioural explanations see (Laibson *et al.*, 1998), Bernheim *et al.* (2001) and Benartzi and Thaler (2013), among others.

[8]Two empirical observations seem robust across countries and specifications. First, household wealth is positively related to the probability of retirement at any age. For example, Imbens *et al.* (2001) show that large lottery gains lead to significant reduction of labour supply, particularly for those around retirement; Brown *et al.* (2010) finds that receipt of inheritance increases probability of retirement especially when inheritance is unexpected. Second, descriptive evidence (Kallestrup-Lamb *et al.*, 2016; Bender *et al.*, 2014) suggests that controlling for wealth, individual earnings are inversely related to the probability of retirement - an observation suggestive of opposing income and substitution effects of earned income on consumption of leisure over the life cycle.

retirement choice and emphasises how the latter emerges as a result of optimal behaviour when labour markets are frictional. It, hence, presents a framework for analysing the relationship between labour-market behaviour, asset accumulation and retirement strategies. Rogerson and Wallenius (2013) discuss the role of non-convexities in the worker's problem, and labour indivisibility in particular, for generating abrupt transitions from employment to non-participation[9]. Our analysis also relies on a non-convexity due to labour indivisibility, but as the latter results from frictions it further implies that workers specialise in work early in life and in leisure later. When labour markets are frictional quits to non-participation are suboptimal if an individual intends to return to employment later.

The model has highly tractable implications for the evolution of wealth distribution in the presence of persistent differentials in earning ability. We employ the 1996 and 2001 panels of the Survey of Income and Program Participation to explore some empirical aspects of the latter. First, we estimate non-parametrically the observed age profiles of the conditional on earnings net worth distributions. We document that wealth gets increasingly dispersed with earnings. The dispersion increases with age and is driven by pronounced lengthening of the right tail of wealth while the bottom quantiles vary little with age. The tenth percentile of wealth increases with age only for households with high observed earnings. We argue that these observations are readily interpretable through the prism of the model. Next we turn to individual households' observed saving outcomes. In the absence of uncertainty the model emphasises the role of persistent differences in earning ability on optimal saving policies. To frame empirical results more closely to the theoretical context we construct permanent earnings proxies and investigate how permanent earnings and wealth map empirically into median household saving outcomes. In order to limit the influence of extreme observations we follow a double stage least absolute deviations estimation (as proposed by Amemiya (1982)).The evidence is suggestive of the model's empirical adequacy in aspects where its

[9]In addition to labour indivisibility, they suggest two different sources of non-convexity - non-linearity of wages with respect to hours worked (to which they attribute the retirement decision in French (2005)) and fixed time and consumption costs

implications differ from traditional life-cycle models.

As documented in Section 5.3[10], while households with low earnings have low net worth on average, households at the left tail of the net worth distribution have high earnings, high levels of education, and are younger than their counterparts in the right tail. In an application of the model we demonstrate how such pattern could emerge as a result of education choice early in life. In particular, we ask how initial endowments of wealth and abilities (productivities prior to obtaining education) relate to the optimality of investing in costly education, given that subsequent behaviour is as in the model. The analysis implies that, given wealth, investment is only optimal for workers of sufficiently high ability; given ability, it is only optimal for workers with sufficiently low wealth. These imply that wealth dynamics induced by costly education early in life provides an explanation for the observed pattern.

The rest of this paper is organised as follows. Section 2 presents the model and formulates the optimal consumption/saving and labour-market strategies for workers of given earning ability. Section 3 analyses how optimal policies differs across workers with different productivity. Section 4 extends the framework by introducing an education choice at the beginning of life and tracks the implications for early-age wealth dynamics. Section 5 presents some empirical results. Section 6 concludes.

2 Model

2.1 Environment

Time is continuous. An infinitely-lived[11] risk-averse worker has productivity w and wealth A. Labour is indivisible. The worker borrows and lends at a constant risk-free rate, r, and derives utility from consumption and leisure. Preferences are additively separable with flow

[10] See Chapter **??** of this thesis for a similar result obtained using a different dataset.

[11] The environment generalises to exogenous Poisson death process but this brings no extra insight.

utility from consumption, $u(c)$, such that $u'(.) > 0$, $u''(.) < 0$, $\lim_{c \to 0} u'(c) = \infty$ and flow utility, u_b, from leisure.

At any point in time the worker is in one of three labour-market states - non-participant, job-searcher or employed. The labour market is frictional as she becomes employed only upon accepting a job offer. When employed she earns w and decides whether to remain employed or quit into non-participation or unemployment. When non-participant she earns no income, enjoys leisure, and decides whether to remain non-participant or transit to unemployment. When searching she earns unemployment income, $b < w$, enjoys no leisure, samples job offers at Poisson rate λ, and decides to remain unemployed or transit to non-participation. For simplicity, assume she faces no layoff risk while employed.

An important assumption is that earned wage, w, is time-invariant[12]. While the analysis generalises to i.i.d. income uncertainty the focus on long-term earning ability is consistent with an emphasis on life-cycle rather than precautionary saving motives. Finally, given flat earning profiles, assume that the rate of time preference equals the real interest rate, implying a taste for flat consumption profiles as well.

Given this environment we now turn attention to workers' optimisation problem.

2.2 Optimisation problem

Let $V^{np}(A, w)$, $V^{js}(A, w)$ and $V^{e}(A, w)$ be the lifetime utilities from non-participation, job search and employment. Further, let $V^{n}(A, w) \equiv \max\{V^{np}(A, w), V^{js}(A, w)\}$ denote lifetime utility from non-employment. The values from working or not given $\{A, w\}$ are described

[12] A constant growth rate in w generates convex regions in the value function for employment, inducing agents to pursue strategies that convexify their payoffs (for example, lotteries) - behaviour from which we abstract.

by the system of Bellman equations (see Appendix A.1)

$$rV^n(A,w) = \max \left\{ \begin{array}{r} \max_{c\geq 0}[u(c) + u_b + V_A^n(A,w)(rA - c)] \\ \\ \max_{c\geq 0}[u(c) + V_A^n(A,w)(rA + b - c) + \\ \lambda \max(V^e(A,w) - V^n(A,w), 0)] \end{array} \right\} \tag{1}$$

$$rV^e(A,w) = \max \left\{ \begin{array}{l} \max_{c\geq 0}[u(c) + V_A^e(A,w)(rA + w - c)] \\ \\ rV^n(A,w) \end{array} \right\} \tag{2}$$

These summarise the discussion of labour-market states and optimal behaviour as stated in Section 2.1. It is immediate that in any state optimal consumption requires that the marginal utility of consumption equals the marginal value of the asset[13]

$$u'(c^i(A,w)) = V_A^i(A,w), \forall i \in \{np, js, e\} \tag{3}$$

Consider an employed worker. Using the (3) and (2), her discounted lifetime utility is

$$rV^e(A,w) = u(c^e(A,w)) + u'(c^e(A,w))(rA + w - c^e(A,w)) \tag{4}$$

Assuming differentiability, total differentiation of (4) with respect to time implies

$$u''(c^e(A,w))c_A^e(A,w)(rA + w - c^e(A,w))^2 = 0 \tag{5}$$

Following the same argument for non-participating workers, optimal consumption and

[13]Without ad-hoc constraints on borrowing a worker will not face liquidity constraints as long as $rA+b > 0$. The equality counterpart to the latter identifies the natural borrowing limit.

wealth dynamics requires

$$u''(c^i(A,w))c^i_A(A,w)\dot{A}^2 = 0, \forall i \in \{np, e\} \tag{6}$$

where $\dot{x} \equiv \partial x/\partial t$. Workers attain perfect consumption smoothing within each spell of employment or non-participation. There are two types of strategies consistent with (6). One possibility is that a worker consumes all her income at every instant ($c^e = rA + w$ or $c^{np} = rA$). If ever optimal, this is optimal forever - wealth remains constant and the worker solves the same problem in every future state. Alternatively, a worker could follow a flat consumption path over the duration of a spell and accumulate/decumulate assets until changing employment state. No other strategies could be optimal according to (6).

Inspection of (1) reveals that perfect consumption smoothing is not optimal for unemployed workers. We postpone the discussion of their optimal strategies until Section 2.4 and first characterise the behaviour of the employed.

2.3 Optimal behaviour of employed

Given (6) one potentially optimal strategy for an employed worker is to work forever and consume all income in perpetuity. Similarly a potentially optimal strategy for a non-participant is to never seek employment but enjoy leisure and consume asset income forever - that is retire permanently. Notice that in both cases a worker consumes her permanent income.

Other potentially optimal strategies involve asset accumulation/decumulation and switches between employment, non-participation and unemployment[14]. Quits into non-participation allow the worker to enjoy leisure immediately; if planning to become employed again, however, she will experience a spell of frictional unemployment. The tension is resolved by a third possible strategy - she works and saves in order to accumulate sufficient wealth to retire

[14]It is easy to see that an employed worker never quits into unemployment. Suppose she does. Then unemployment is preferred to both employment and non-participation. Then (1) implies that the value from unemployment is identical to the value of employment at wage $b < w$. Standard arguments imply that the value of employment is increasing in w which is a contradiction.

in future. Formally, consider three possibly optimal labor-market strategies for an employed worker given $\{A, w\}$:

Strategy 1: (Work forever) Work and consume permanent income, $c(A, w) = rA + w$. If ever optimal, this is optimal forever. The associated lifetime payoff is $\Pi^E(A, w) = u(rA + w)/r$.

Strategy 2: (Permanently retire) Never participate and consume permanent income, $c(A, w) = rA$. If ever optimal, this is optimal forever. The associated lifetime payoff is $\Pi^R(A, w) = (u(rA) + u_b)/r$.

Strategy 3: (Optimal retirement plan) Work, consume permanent income[15] $c(A, w) < rA + w$ and save. Once a threshold amount of wealth, $A^R(w)$, is accumulated, retire permanently and consume $c(A, w) = rA^R(w)$ forever after.

The rest of this section demonstrates that given any initial $\{A, w\}$ one and only one of these strategies is optimal. As a starting point, consider the optimal behaviour of a worker pursuing Strategy 3.

2.3.1 Characterisation of the optimal retirement plan

By construction strategies 1 and 2 are consistent with optimal consumption dynamics described by (6). Consider a worker pursuing strategy 3. While employed she saves. Then (6) requires that consumption is constant for the duration of the employment spell. Let $c^*(w)$ denote its optimal level. Once the worker accumulates a stock of wealth $A^R(w)$ she retires and consumes $rA^R(w)$ forever. Optimal consumption smoothing and separability between consumption and leisure imply $c^*(w) = rA^R(w)$.

Let $\tau(A, A^R)$ be the optimal time to retirement given current wealth and wealth at retirement[16]. Solving the wealth accumulation equation $\dot{A} = rA + w - rA^R$ forward from 0

[15] The discussion of what permanent income is when a worker chooses the span of working life is postponed until Section 2.3.1.

[16] Time to retirement depends on w but the argument is suppressed for brevity here as well as in the expressions for lifetime payoffs derived below.

to $\tau(A, A^R)$ yields

$$\begin{aligned}
\int_0^{\tau} \frac{\dot{A}}{rA + w - rA^R} dt &= \int_0^{\tau} dt \\
\frac{1}{r} \ln(rA + w - rA^R)\Big|_0^{\tau} &= \tau \\
\tau(A, A^R) &= \frac{1}{r} \ln\left(\frac{w}{rA + w - rA^R}\right) \qquad (7)
\end{aligned}$$

Time to retirement decreases with wealth. In the limit as A approaches A^R, τ approaches zero. As rA approaches $w - rA^R$ time to retirement approaches infinity.

A worker earns labour income only in annuity until retirement. Given A^R and (7) the discounted value of future labour income is

$$\begin{aligned}
w \int_0^{\tau(A,A^R)} e^{-rt} dt &= \frac{w}{r}\left(1 - e^{-r\tau(A,A^R)}\right) \\
&= \frac{c^*(w) - rA}{r}
\end{aligned}$$

and after rearranging

$$\frac{c^*(w)}{r} = A + w \int_0^{\tau} e^{-rt} dt \qquad (8)$$

(8) reveals that when planning for retirement workers consume permanent income. Given τ a worker behaves identically to a pure life-cycle consumer who faces the same time to retirement exogenously and is subject to an exogenous stream of earnings of the same present value.

The worker consumes c^* in perpetuity and after time $\tau(A, A^R)$ enjoys leisure in perpetuity. Therefore her lifetime payoff is

$$u(c^*) \int_0^{\infty} e^{-rt} dt + u_b \int_{\tau(A,A^R)}^{\infty} e^{-rt} dt$$

Integrating and substituting (7), the payoff from pursuing strategy 3 in terms of A^R is

$$\Pi(A, A^R) = \frac{u(rA^R)}{r} + \frac{rA + w - rA^R}{rw} u_b \tag{9}$$

Maximizing (9) with respect to A^R yields the familiar first-order condition for optimal consumption-leisure choice:

$$u'(c^*) = \frac{u_b}{w} \tag{10}$$

The worker chooses A^R (and, equivalently c^*) in order to equalise the marginal rate of substitution of consumption for leisure to her earned income. At the margin by delaying retirement for an instant she loses the opportunity to enjoy an immediate flow of leisure, u_b, but is able to gain an extra flow of income w which increases her utility from consumption by $wu'(c^*)$.

Consumption during saving for retirement is increasing in w/u_b. All else equal, workers with strong preference for leisure consume less and accumulate wealth faster so they are able to enjoy leisure sooner in the future. Similarly high-wage workers consume more during employment and the subsequent spell of retirement.

Using (9) the payoff from strategy 3 in terms of the model parameters is

$$\Pi^P(A, w) = \frac{u(c^*(w))}{r} + \frac{rA + w - c^*(w)}{rw} u_b \tag{11}$$

Let $\underline{A}^E(w) \equiv (c^* - w)/r$. If $A < \underline{A}^E(w)$ strategy 3 is not feasible as $\dot{A}_{|A<\underline{A}^E} = rA + w - c^* < 0$, i.e. workers decumulate wealth if consuming c^*. Let $\bar{A}^E(w) \equiv A^R(w)$.

2.3.2 Optimal consumption during employment

While easy to see that strategies 1-3 are consistent with (6), they need not be either optimal or the only optimal strategies. The next result demonstrates their optimality among a set of solutions characterised by the following property (later verified to be consistent with the prescribed optimal behaviour):

Property 1. *Whenever it is optimal for an employed worker to quit into non-employment, she retires permanently.*

Assuming Property 1 allows us to temporarily disregard strategies involving cycles between the three labour market states and implies the following result:

Proposition 1. ***Optimal consumption of employed workers and retirement***
Conditional on Property 1

(i) *If* $A \leq \underline{A}^E(w)$*, an employed worker's optimal strategy is to work forever and consume permanent income. Her lifetime utility is* $V(A, w) = \Pi^E(A, w)$*.*

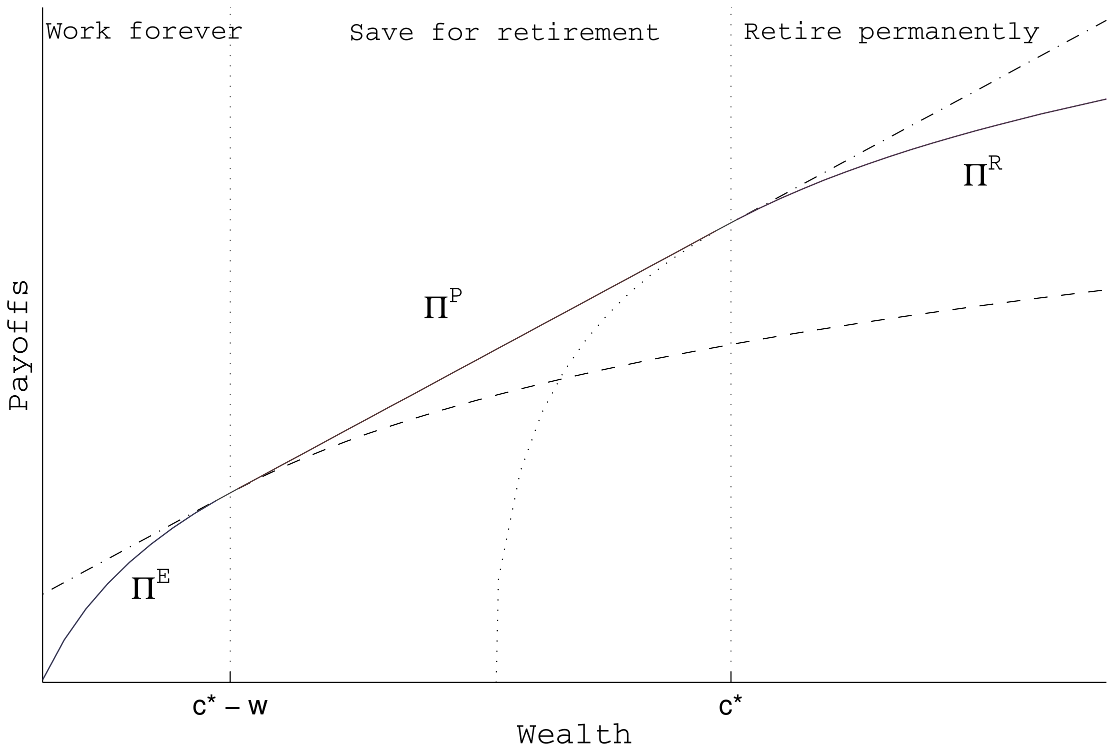
(ii) *If* $A \in (\underline{A}^E(w), \bar{A}^E(w))$*, an employed worker's optimal strategy is to save for retirement and consume permanent income. Her lifetime utility is* $V(A, w) = \Pi^P(A, w)$*.*

(iii) *If* $A \geq \bar{A}^E(w)$*, the worker's optimal strategy is to retire permanently and consume permanent income. Her lifetime utility is* $V(A, w) = \Pi^R(A, w)$*.*

Proof. The result is proved directly. Start with a conjecture: $V(A, w)$ as stated above solves the workers' Bellman equations. Verify that under this conjecture optimal consumption is as stated. Finally verify that under this optimal consumption the choice of $V(A, w)$ is consistent with the Bellman equations. Workings are demonstrated in Appendix A.2. □

Sufficiently asset-poor workers have high value from earned income and do not plan to retire. The asset-rich receive large interest income which can finance consumption while they also enjoy leisure. Between the extreme cases workers save for retirement. The payoffs from Strategies 1 (work forever) and 2 (retire permanently) are increasing and concave functions of wealth but the payoff from pursuing either Strategy 1 or 2 is non-concave. The payoff from optimally saving for retirement is linear in wealth and identifies the convex envelope of

Figure 1: Wealth and labor market strategies



the payoffs from working forever and retiring (Figure 1). When labour is supplied indivisibly at the cost of foregone leisure, saving for retirement convexifies the payoffs from the "pure" actions of work and retirement, allowing workers to achieve an optimal combination of leisure and consumption over the life cycle.

2.4 Optimal behaviour of non-employed and solution

The validity of Proposition 1 relies on Property 1 which is a conjecture about the behaviour of non-employed workers. To complete the solution we characterise the latter and show that Property 1 is indeed a feature of the model. To keep a clear focus on wealth accumulation, the analysis of non-employed workers' behaviour is delegated to Appendix A.3, and only the main results and intuitions are listed here. Optimal behaviour of non-employed workers is fully characterised by the following

Proposition 2. ***Optimal consumption of non-employed workers***

Conditional on Property 1 two wealth levels $\underline{A}^U < \bar{A}^U$ exist such that

(i) A non-employed worker with $A \in [-b/r, \underline{A}^U]$ seeks employment and dissaves. Optimal consumption and savings dynamics is described by the system

$$\begin{aligned} V_A^n(A) &= u'(c^n(A)) \\ c_A^n(A) &= \frac{\lambda(u'(c^n(A)) - u'(c^e(A))}{u''(c^n(A))(rA + b - c^n(A))} \end{aligned}$$

with terminal conditions $c^n(-b/r) = 0$ and $V^n(-b/r) = \frac{u(0)+\lambda V^e(-b/r)}{r+\lambda}$. This regime ends at $\underline{A}^U$ where $S(\underline{A}^U) = u_b$.

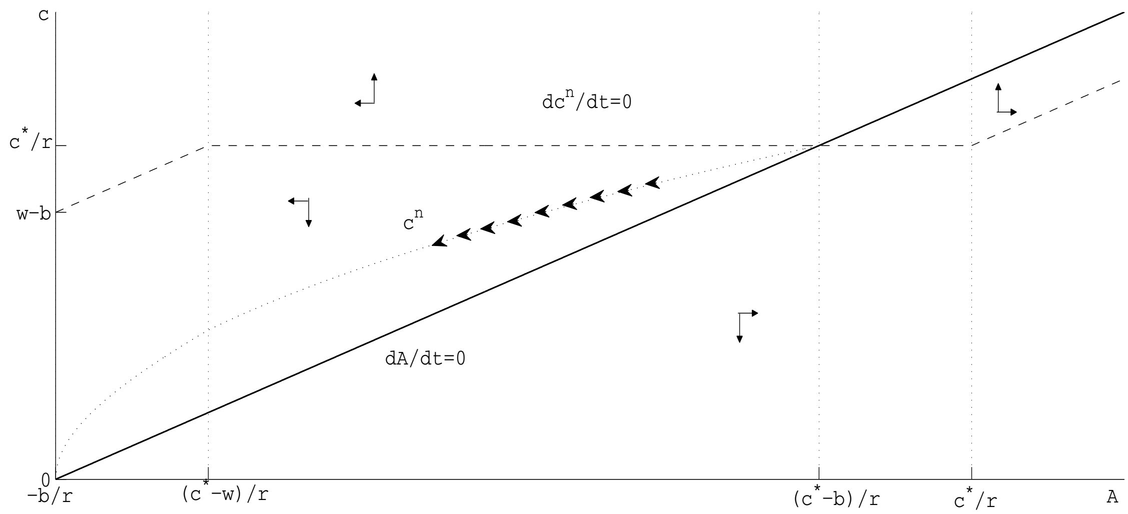
(ii) A non-employed worker with $A \in (\underline{A}^U, \bar{A}^U)$ is non-participant, consumes $c^n(\underline{A}^U)$, and dissaves. This regime ends at $\bar{A}^U$ where $\bar{A}^U = c^n(\underline{A}^U)/r$.

(iii) A non-employed worker with $A \geq \bar{A}^U$ retires permanently, consumes rA, and saves 0.

Proof. The validity of 2 follows by construction from the arguments in Appendix A.3. □

The main arguments behind the result are as follows. Non-employed workers can freely change state between unemployment and non-participation. A worker sufficiently close to the natural borrowing limit relies on unemployment income and future employment prospects to prevent unsustainable debt accumulation, hence optimally avoids leisure. For wealth levels between the borrowing limit and $\underline{A}^U$, unemployment dominates non-participation. The tension between desire for smooth consumption and constraint on borrowing implies a unique saddle path solution characterised by a steady decline of consumption and dissaving (Figure 2). At wealth levels between $\underline{A}^U$ and $\bar{A}^U$ the worker postpones job-search, pursues leisure and dissaves until unemployment dominates again. At sufficiently high wealth, the perspective of an employment opportunity far into the future is discounted sufficiently so that the worker specialises in enjoying leisure and retires.

Figure 2: Phase diagram



The phase portrait only applies for $A \in [-b/r, \underline{A}^U)$.

Figure 3 illustrates the dynamics of consumption and wealth for employed and non-employed workers over the wealth distribution's support. Unlike employed and non-participating workers, the unemployed do not attain perfect consumption smoothing. As they approach the borrowing limit the probability of not finding employment before accumulating unsustainable debt increases. As a result they limit the pace of dissaving due to precautionary motives.

The next result completes the solution by affirming that the behaviour described by Propositions 1 and 2 implies Property 1.

Theorem 1. *The solution to Bellman equations* (1) *and* (2) *implies:*

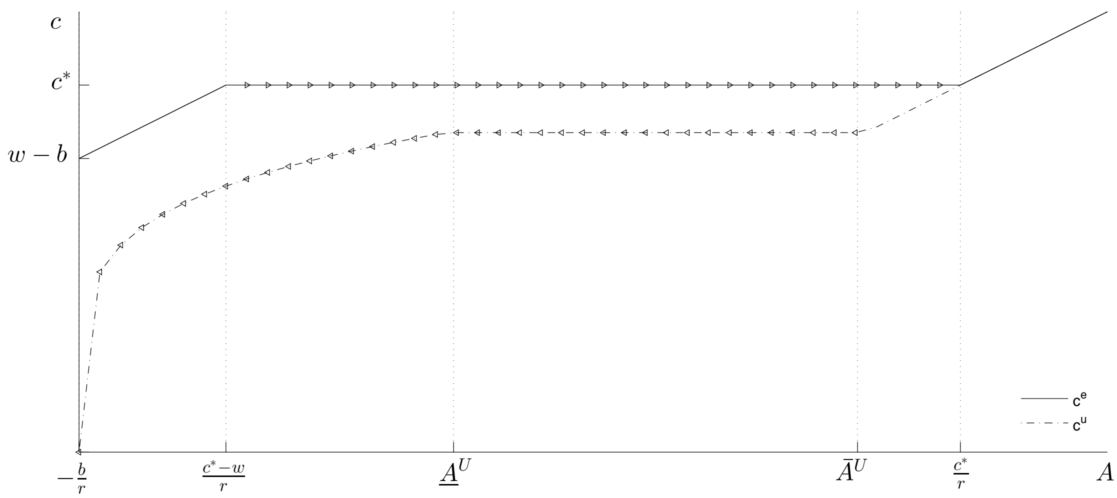
(i) The value function for employed workers, $V^e(A, w)$, is as identified by Proposition 1.

(ii) The value function for non-employed workers, $V^n(A, w)$, is as identified by Proposition 2.

(iii) Property 1 holds.

Proof. Appendix A.4. □

Figure 3: Consumption



The model implies optimal labour-market behaviour with natural life-cycle interpretation and endogenous retirement decision. As rigidities prevent workers from continuously adjusting their labour supply, they achieve optimal consumption-leisure tradeoffs over the life cycle. They choose to work early in life so that they have sufficient assets to retire later. While valuing leisure, they avoid temporary quits into non-participation as finding employment involves costly search. When choosing consumption they simultaneously plan retirement and retire where the marginal value of extra earned income just equals the value of leisure. Consumption/saving decisions and retirement plans depend on wealth endowment and long-term earning ability. In the limit, the most asset-poor workers of given earning ability do not save.

The analysis demonstrates that, qualitatively, endogenous retirement provides an explanation for the well documented empirical fact of low wealth being an "absorbing state" (Hubbard *et al.*, 1994a), while not deviating fundamentally from the main life-cycle abstractions. The next section turns attention to the model's implications for the relationship between long-term earnings and saving behaviour.

3 Earnings heterogeneity

Within this framework, earnings have two opposing effects on retirement plans. The income effect of higher earnings, as a wealth transfer, does not affect consumption but encourages pursuit of leisure (earlier retirement). However, high earnings imply high opportunity cost of leisure (by retiring early a worker forgoes more income and consumption) and the substitution effect results in higher consumption and postponement of retirement. To see how the two effects interact it is more convenient to work with the total effect on consumption. Implicitly differentiating $c^*(w)$ in (10) with respect to w and rearranging

$$\frac{\partial c^*(w)}{\partial w}\frac{w}{c^*(w)} = -\frac{u'(c^*)}{u''(c^*)c^*} \tag{12}$$

which links the elasticity of consumption with respect to earnings depends on the elasticity of intertemporal substitution/risk aversion properties of preferences. For future references, let $R(.)$ denote the coefficient of relative risk aversion. Recall that $\bar{A}^E(w) = c^*(w)/r$ identifies the upper bound of the wealth distribution for employed workers of ability w, or equivalently their wealth at retirement. It is strictly increasing in w with the rate of increase depending on the risk-aversion properties of the utility function. $\underline{A}^E(w) = (c^*(w) - w)/r$ identifies the lower bound below which workers do not save. The relationship between $\underline{A}^E$ and w could be increasing, decreasing or non-monotonic, depending on preference parameters. The range of wealth levels where workers save, $\bar{A}^E - \underline{A}^E = w/r$, is proportional to earnings.

Consider a set of workers facing the same time to retirement, $\bar{\tau}$. By (7)

$$e^{-r\bar{\tau}} = \frac{rA - c^*(w) + w}{w}$$

and after rearranging

$$A|_{\tau=\bar{\tau}} = (1 - e^{-r\bar{\tau}})\underline{A}^E + e^{-r\bar{\tau}}\bar{A}^E$$

The locus of $\{A, w\}$ pairs where workers face the same retirement horizon is a weighted mean of the $\underline{A}^E$ and $\bar{A}^E$ loci. The weight equals the discount factor for time $\bar{\tau}$ into the future. Figure 4 illustrates the latter by mapping the plane into times to retirement for different values of the risk aversion coefficient, all assuming isoelastic utility[17]. The two effects cancel out exactly when preferences for consumption are described by log-utility so that all workers with zero wealth face the same time to retirement. As a result scaling up earnings scales permanent income proportionally and homothetic saving policies obtain. If instead $R(c) > 1$, time to retirement relates to earnings non-monotonically but starts decreasing after a threshold. Above the threshold, higher earnings translate to less than proportionate increases in permanent income as workers plan to retire sooner. The opposite occurs when relative risk aversion is below unity.

More precisely, recall the accumulation equation for employed savers

$$\dot{A} = rA + w - c^*(w) \tag{13}$$

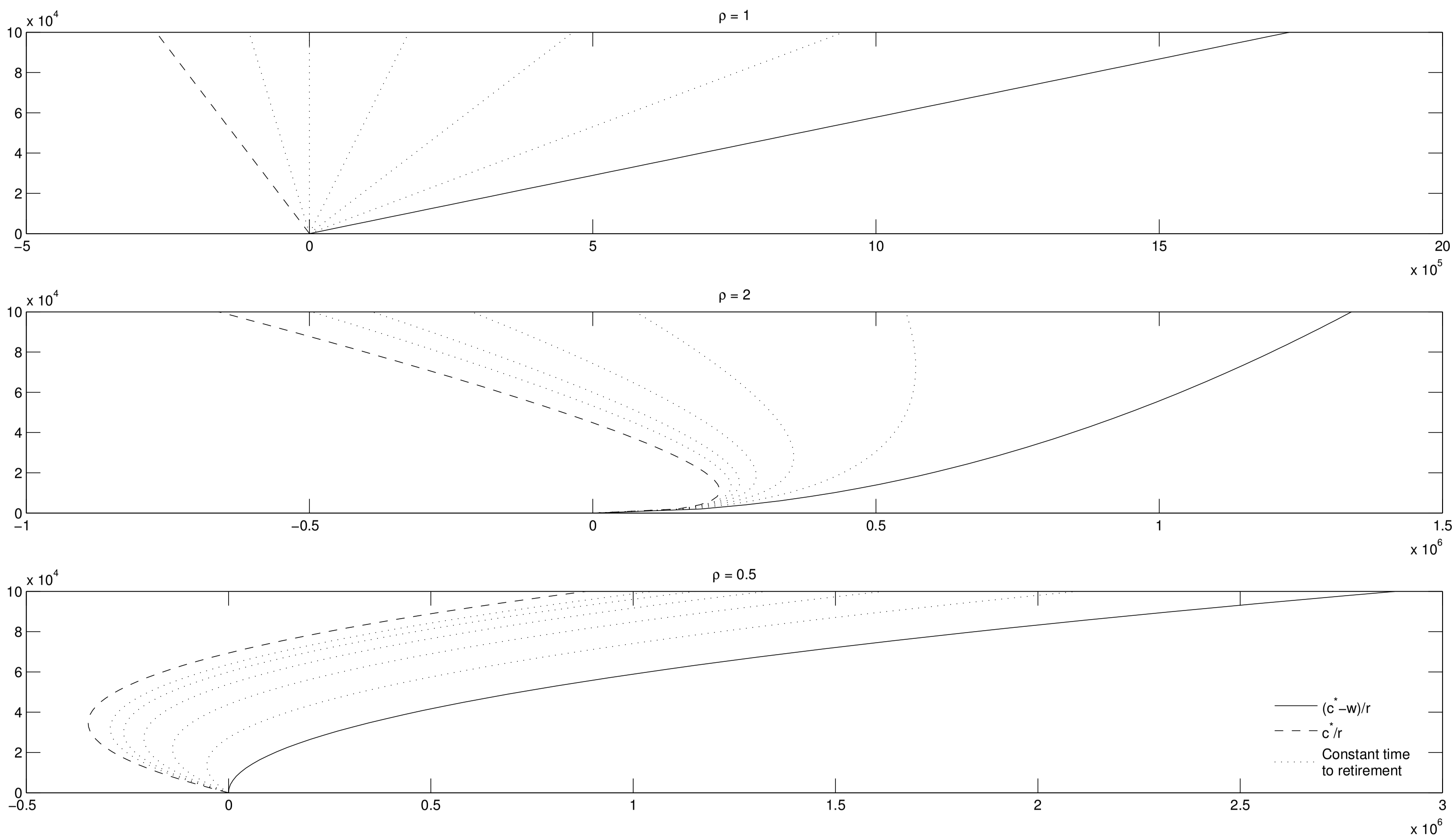
Consider a set of workers of different w optimally choosing to save the same amount $\bar{\dot{A}}$. Rearranging (13), these workers have stock of wealth

$$A|_{\dot{A}=\bar{\dot{A}}} = \frac{\bar{\dot{A}}}{r} + \underline{A}^E(w)$$

This identifies an "iso-saving" set for $\bar{\dot{A}}$. It is geometrically represented in the (A, w)-plane as a horizontal translation of the $\underline{A}^E$ locus by distance $\bar{\dot{A}}/r$. Figure 5 plots some iso-saving curves in the case of isoelastic utility for three benchmark values of the risk aversion coefficient. Consider "active" saving rates out of earned income, $(w - c^*(w))/w$.

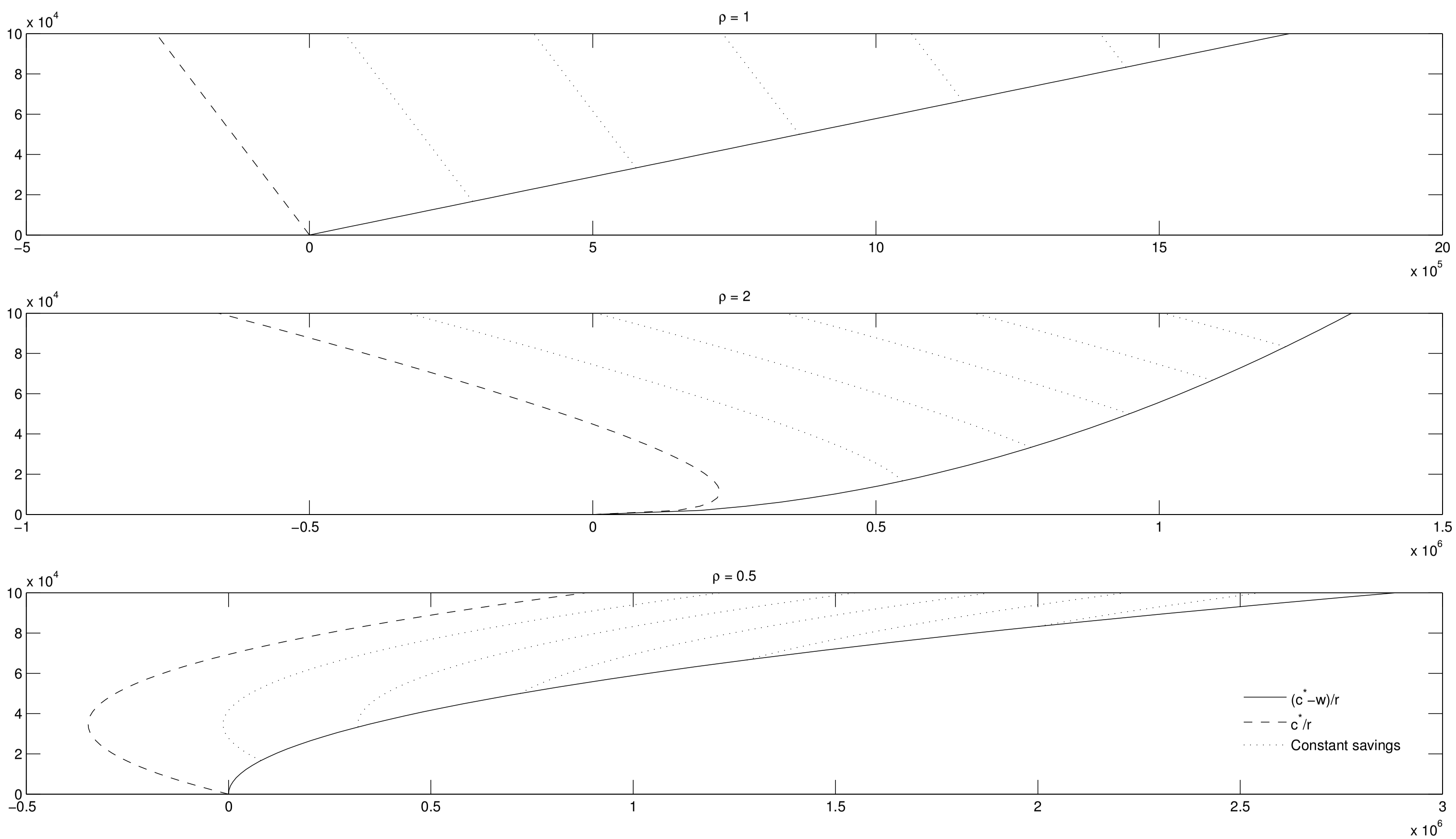
[17]Figures 5 and 4 are plotted for $r = 0.05$ and u_b set to imply (somewhat arbitrarily) that a worker with zero wealth earning 60000 per year retires in exactly 40 years. ρ stands for the coefficient of relative risk aversion.

Figure 4: Time to retirement



The dotted curves track pairs of wealth and earnings implying constant time to retirement. The leftmost line identifies 50 years to retirement, while the rightmost identifies 10 years to retirement.

Figure 5: Savings



The dotted curves track pairs of wealth and earnings implying constant savings.

Differentiating with respect to w and rearranging

$$\frac{\partial[(w - c^*)/w]}{\partial w} = \frac{c^*(w)}{w^2}\left[1 - \frac{1}{R(c^*)}\right]$$

Active saving rates increase with earnings when $R > 1$ and are independent when $R = 1$. Similarly for "total" saving rates out of earned income, $\dot{A}/w$,

$$\frac{\partial(\dot{A}/w)}{\partial w} = \frac{r}{w^2}\left[\left(1 - \frac{1}{R(c^*)}\right)A^R - A\right]$$

Total saving rates depend on risk aversion and on the distance between a worker's current wealth and their target retirement wealth.

To summarize the discussion, under endogenous retirement optimal saving behaviour depends on retirement plans. The relationship between saving rates and earnings depends on how workers substitute consumption for leisure. If the income effect on leisure dominates, high earners retire earlier hence save more than proportionately in comparison to low earners; the opposite occurs if the substitution effect dominates. The model presents a highly tractable description of wealth dynamics in the presence of heterogeneity in permanent earnings and wealth, implying the following. Asset-poor workers accumulate wealth slowly and remain in the left tail of the distribution for long time; the asset-rich accumulate wealth quickly until retirement; the relationship between total saving and wealth is linear. High earners retire with more assets and are found over a broader support of wealth; they save more or less than proportionately in comparison to low earners depending on the interaction between income and substitution effects of earnings. The empirical relevance of these predictions is discussed in Section 5, while Section 4 turns attention to the optimal choice of education and its implications for the distribution of wealth and earnings.

4 Education choice

Investment in education allows individuals to expand their long-term earning prospects early in life but involves significant costs that could largely affect subsequent life-cycle outcomes. This section employs the above framework to explore the optimality of educational investment when individuals also plan retirement efficiently.

More concretely, suppose that at the beginning of time a worker with ability w_0 and wealth A_0 can choose to invest in education or not[18]. For simplicity the choice is binary (interpreted as pursuit of a higher degree) and acquisition is instantaneous. Education costs a fixed fee, k, and increases productivity. In particular, suppose that a worker with ability w_0 earns w_0 without education and $w_0(1+e)$ with education. For tractability assume that following the decision the worker becomes employed immediately, hence precautionary motives do not affect the choice[19].

Let $V_0(A_0, w_0) \equiv V^e(A_0, w_0)$, $V_1(A_0, w_0) \equiv V^e(A_0 - k, w_0(1+e))$, $c_0(A_0, w_0) \equiv c^e(A_0, w_0)$, $c_1(A_0, w_0) \equiv c^e(A_0 - k, w_0(1+e))$, $\tau_0(A_0, w_0)$ and $\tau_1(A_0, w_0)$ denote the lifetime payoffs, optimal consumptions and times to retirement to a worker with $\{w_0, A_0\}$ if investing (state 1) or not (state 0) in education. Given Proposition 1 and (7)

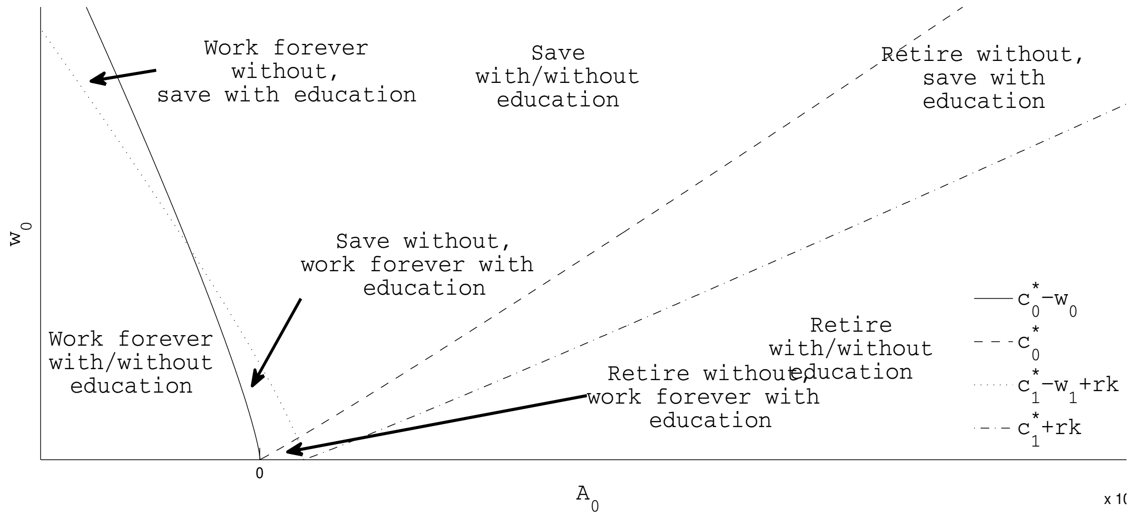
$$
\begin{aligned}
rV_0(A_0, w_0) &= u(c_0(A_0, w_0)) + e^{-r\tau_0(A_0, w_0)} u_b \\
rV_1(A_0, w_0) &= u(c_1(A_0, w_0)) + e^{-r\tau_1(A_0, w_0)} u_b
\end{aligned}
$$

As the worker becomes employed immediately a degree is pursued if and only if $V_1(A_0, w_0) \geq V_0(A_0, w_0)$. The decision maps ability and wealth endowment into a productivity level and initial wealth for the consumption problem. Notice that the endowment $\{w_0, A_0\}$ pins down the optimal labour-market strategies conditional on investing in education or not (see Figure

[18] As demonstrated later, postponing educational choice has no option value in this setting. Even if the choice was available later in life, workers would still invest when young.

[19] This is a strong assumption but simplifies the analysis considerably, as it allows us to abstract from the behaviour of unemployed workers near the natural borrowing limit and its inductive implications over the whole support of wealth. Thus the analysis emphasises life-cycle motives in isolation.

Figure 6: Optimal labor market strategies conditional on education



6).

A worker is indifferent to education if and only if

$$V^e(A_0, w_0) = V^e(A_0 - k, w_0(1 + e)) \tag{14}$$

or equivalently

$$u(c_1(A_0, w_0)) - u(c_0(A_0, w_0)) = (e^{\tau_0(A_0,w_0)} - e^{\tau_1(A_0,w_0)})e^{-r}u_b$$

that is if the gain in consumption (in utility terms) just equals the loss of utility due to possibly delaying retirement. The set of abilities and wealth endowments where indifference obtains is henceforth referred to as the "education efficiency frontier". Appendix B constructs the frontier by analysing the decision of a worker based on her ex-ante (without education) and ex-post (with education) labour market strategies but the main intuition and results are discussed here.

At sufficiently low wealth, individuals work forever irrespective of educational attainment. They only trade consumption possibilities and are always just willing to substitute a unit of wealth for r extra units earnings. The benefit from education is a perpetual stream of extra earnings. At the margin this is compared in present value terms to the one off cost of education incurred at the time of the decision. Workers of sufficiently high ability enjoy higher extra earnings because education complements ability. This implies a threshold wealth-independent ability where education becomes optimal and gives rise to a flat region of the education efficiency frontier.

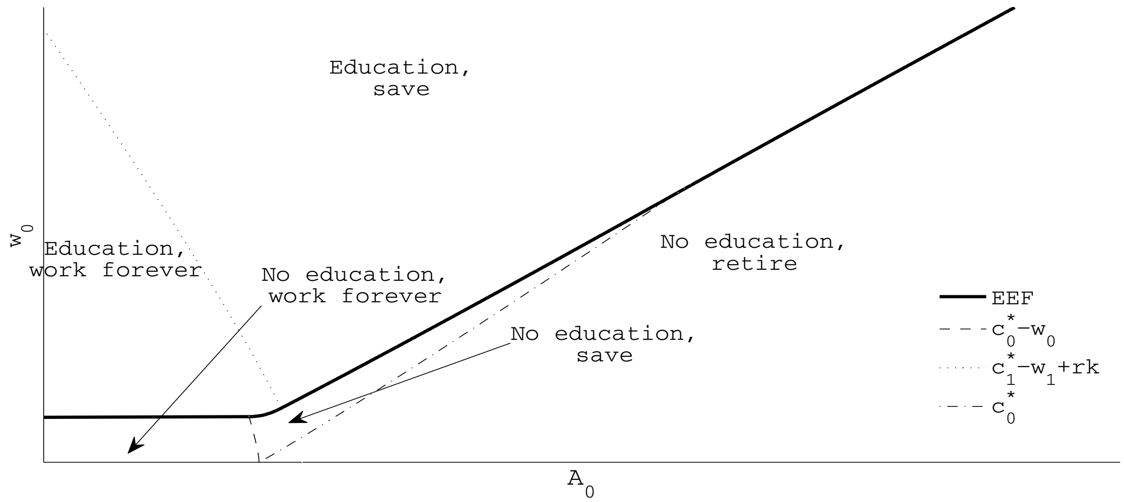
Workers with larger wealth endowment save for retirement. They enjoy boosted earnings only in annuity. The higher their wealth endowment, the sooner they retire - higher earnings translate to lower increases in permanent income and consumption. They willingly substitute a unit of wealth for $r \times (w_0)/(rA^R - rA) > r$ extra earnings. The threshold ability where indifference obtains therefore increases with the wealth endowment. In the limit, individuals sufficiently wealthy to retire immediately both with and without education, have no value from obtaining a degree - no one obtains education to retire immediately.

These arguments imply[20] that the education efficiency frontier is a continuous, (weakly) upward-sloping schedule in the (A_0, w_0)-plane (Figure 7). Given any wealth endowment, investment is optimal only for workers of sufficiently high ability; given ability investment is optimal only for sufficiently wealth-poor workers. This further demonstrates that given the stationary nature of the decision problem the assumption of educational investment being available only early in life is without loss of generality - as they accumulate wealth during employment workers of any ability lose value by postponing investment. Education is pursued early in life as when wealth is low, retirement is far in the future, and higher earnings result in larger increases in permanent income.

As a result high-earning workers start employment with less assets than low earners with identical pre-education wealth endowment, who find educational investment unprofitable. To the extent that education is sufficiently expensive, it is a reason for significant wealth decumulation early in life, for those who invest. This implies that the left tail of the marginal wealth distribution is populated not by the income poor, but by young educated high earners,

[20]See Appendix B for a more thorough treatment.

Figure 7: Education efficiency frontier and optimal strategy



as documented empirically in Section 5.3.

5 Empirical analysis of wealth dynamics

While highly stylized the model combines the main abstraction of life-cycle theory with the view of endogenous retirement, and suggests a tractable description of wealth dynamics emphasising pure life-cycle motives and persistent differences in earnings. This section presents some descriptive evidence for the evolution of wealth of households with different earnings from the 1996 and 2001 panels of the Survey of Income and Program Participation. The analysis focuses on the household, rather than the individual as unit for analysis and selects a sample of middle-aged households that are unlikely to be in transitory stages of their life. We start by describing the estimation sample, and proceed to investigate how wealth and long-term earning ability map empirically into median saving outcomes. Finally, we document the age profile of the conditional on observed earnings net worth distribution, and demonstrate that a calibrated version of the model is able to account for the documented facts. We conclude that, while stylized, the model presents a data-consistent description of households' life-cyclical wealth dynamics.

5.1 Data and summary statistics

The SIPP is a US household-based survey running since 1984. It comes in a series of 3-to-4-year panels, each featuring a different nationally representative sample of households. A household is interviewed once every four months and upon the interview data on demographic, income and employment-related (among others) outcomes are collected for each household member retrospectively at monthly level over the latest four months. Comprehensive data on the stocks of assets and liabilities at household level is collected once a year during the interviews taking place in the third, sixth, ninth and twelfth waves[21](The 1996 panel consists of 12 waves, while the 2001 panel only consists of 9.). The first round of interviews for the 1996 (2001) panel took place between August and November 1995 (February 2001 and June 2001). Wealth data was first collected between March and June 1996 (October 2001 and January 2002). At this stage 27120 (22099) households were interviewed. Attrition resulted in only 22438 (20026) of these being followed up until the last round of interviews.

Given the focus on long-term wealth dynamics, using the full panel sample for the analysis is inappropriate. First, about 30% of the households are non-family and some family households contain no spouse. Further, the composition of some family households changes across the waves. Associating reported net worth with the stock of accumulated life-cycle resources relevant for decisions in these cases is flawed. Second, availability of wealth data at only annual frequency suggests that the analysis should also take place at annual frequency. Constructing consistent annual earnings from monthly observations requires that households are interviewed continuously during the length of the panel. Third, households where the head or spouse is retired or in full-time education are likely to be in transitory phases of their life with respect to saving decisions. In view of these, the sample is restricted

[21]See Czajka *et al.* (2003) for a comprehensive comparison of the wealth information in the SIPP and the other two major household surveys - PSID and SCF. While the SCF is widely considered to contain the most comprehensive and reliable data on wealth, it is a cross sectional survey. On the other hand, while the PSID has very long panel, in the few releases where wealth data is collected it excludes assets in pension accounts which we perceive as fundamental for life-cycle considerations.

Table 1: Descriptive statistics, SIPP and sample for estimation

	1996		2001	
	SIPP	Sample	SIPP	Sample
Demographics				
Age, head	49.817	43.239	50.340	43.914
	(16.947)	(7.317)	(16.850)	(7.451)
White, head	.843	.895	.833	.881
	(.364)	(.306)	(.372)	(.323)
Retired, any	.292	0	.287	0
	(.455)	(0)	(.453)	(0)
Enrolled, any	.053	0	.047	0
	(.224)	(0)	(.211)	(0)
Household				
Family	.699	1	.693	1
	(.458)	(0)	(.461)	(0)
Married	.548	1	.544	1
	(.498)	(0)	(.498)	(0)
Female, head	.459	.297	.476	.346
	(.498)	(.457)	(.499)	(.476)
Education, head				
< *High school*	.199	.105	.167	.089
	(.399)	(.307)	(.373)	(.285)
High school	.292	.286	.291	.265
	(.455)	(.452)	(.454)	(.441)
< *Degree*	.280	.293	.289	.300
	(.449)	(.455)	(.453)	(.458)
Degree +	.229	.317	.254	.346
	(.420)	(.465)	(.435)	(.476)
Wealth/Earnings				
Earnings	33676.94	59682.43	36217.97	64201.19
	(39559.54)	(43952.64)	(40952.39)	(46521.17)
Income	41571.06	65805.93	44097.34	67532.04
	(38521.82)	(47156.46)	(39742.95)	(48086.64)
Net worth	117452.7	122067.2	144091.5	160955.0
	(507787.7)	(290673.4)	(560610.7)	(284907.2)
Observations	27120	4946	22099	4780

to married family households, with the same household head and spouse over the panel duration, interviewed in every wave of the survey, both the household head and the spouse are between 30 and 59 years old and neither is enrolled in full time education or retired[22]. The subsample used in what follows thus contains 4946 households from the 1996 panel (4780 households from the 2001 panel), observed annually for four (three) consecutive years. Table 1 presents some descriptive statistics for the full SIPP sample (from the cross-section of observations at the time of the third wave interview, when asset data was first collected) and the sample for estimation[23]. Unsurprisingly (e.g. see Table 1 in Alan *et al.* (2015)), the two samples represent distinct populations. On average, households in the estimation sample are younger, more highly educated, wealthier, and earn more than those in the full SIPP sample. Earnings represent the overwhelming share of their total income.

Next, turn attention to the cross sectional distribution of net worth and earnings. To summarize two aspects of these, we estimate non-parametrically the distribution of net worth reported in the first wave of the survey conditional on average annual earnings over the sample period, as well as the distribution of annual earnings conditional on net worth[24]. Figure 8 presents contour plots for the estimated conditional CDFs with the curves on the graph representing the quantiles of the conditional distribution functions.

The top panels of the figure plot the quantiles of net worth conditional on earnings. Net worth is highly disperse and its dispersion increases with earnings. Up to the middle 80 percent, the distribution for high-earners dominates the one for low-earners. Top quantiles expand significantly while the 10th percentile is nearly constant over the distribution of earnings.

The bottom panels plot the quantiles of earnings conditional on net worth. Households

[22]This sampling decision is consistent with prior studies (e.g. see Dynan *et al.* (2004) and Alan *et al.* (2015)).

[23]Henceforth, all income and wealth related quantities are reported in real terms after being deflated using CPI.

[24]The estimation uses the method of Li and Racine (2008), and employs Epanechnikov kernels and data-driven bandwidth selection. The procedure favours bandwidths close to 10000 real dollars for both variables but the results are robust under moderate deviations from this. The reasons for using such specification are briefly discussed below.

Figure 8: Distributions of net worth and earnings

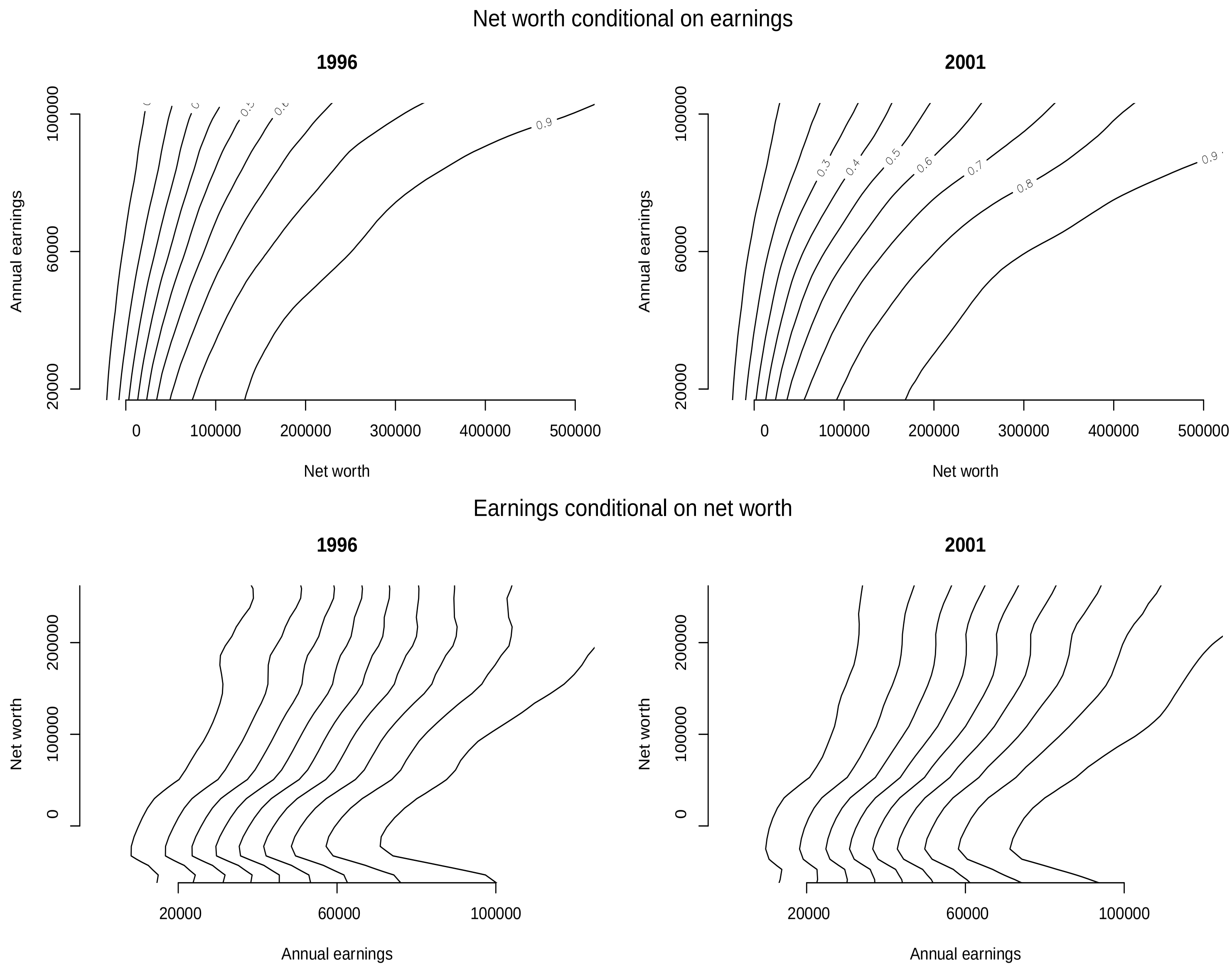


Table 2: Summary statistics by net worth

Net worth	< -12561	$[12561, -529)$	$[-529, 716)$	$[716, 17233)$	$[17233, 76850)$	$[76850, 198050)$	≥ 198050
Net worth quantiles	$[0, 0.05)$	$[0.05, 0.1)$	$[0.1, 0.12)$	$[0.12, 0.25)$	$[0.25, 0.5)$	$[0.5, 0.75)$	$[0.75, 1]$
Earnings	47248	40128	28017	39540	51907	67833	96769
Age	41.19	40.88	41.20	41.33	42.36	44.45	46.31
Education, head							
< *High school*	0.0962	0.175	0.271	0.217	0.104	0.0394	0.0234
High school	0.347	0.308	0.375	0.337	0.322	0.266	0.136
< *Degree*	0.318	0.321	0.260	0.282	0.346	0.334	0.226
Degree +	0.238	0.196	0.0938	0.165	0.228	0.361	0.615
Education, spouse							
< *High school*	0.130	0.142	0.281	0.243	0.120	0.0511	0.0209
High school	0.377	0.354	0.427	0.346	0.342	0.308	0.168
< *Degree*	0.243	0.346	0.104	0.270	0.305	0.319	0.257
Degree +	0.251	0.158	0.188	0.141	0.233	0.321	0.554

Households are grouped into 7 net worth categories. Two equally large groups span the bottom 10 percent of the sample. The third group spans 2 percent of the sample close to zero net worth. The fourth group complements the sample to the 25th percentile. The top three groups are equally large and each corresponds to 25 percent of the sample. Age is average age of household head and spouse.

around zero net worth earn least on average, in particular, less than households in net debt[25]. To explore this more closely, Table 2 reports the average earnings, age, and educational attainment of household heads and spouses for households grouped in seven categories increasing in net worth. The results for earnings are consistent with the pattern identified by the bottom panel of Figure 8. Households within the bottom 25 percent of the sample are similar in terms of age, and average age increases across net worth quartiles. Average educational attainment tracks closely the pattern in earnings. Households in the left tail of the distribution are not only high earners relative to those at zero net worth, but also more highly educated. The 5 percent of households in the left tail have educational attainment comparable to the second quartile, yet they are somewhat younger and earn less. Households around zero net worth have the worst educational record and are predictably the lowest earners. The pattern implies that high earning households incur costly expenditures early in life unlike low-earners of similar wealth. As demonstrated in Section 4, costly education provides an explanation for this observation.

5.2 Dynamics of wealth over the distribution of earnings

This section describes the empirical relationship between observed household saving outcomes on one hand, and permanent earnings, net worth and age, on the other. The purpose of the exercise is twofold. First, it seeks to assess the adequacy of the model's implications for wealth dynamics, summarized in Figure 5, in reduced form, when permanent earnings are appropriately accounted for. To the extent that the model is seen as a description of data-generating process, reduced-form estimation of (13) is informative of the models broader adequacy. Further, wealth dynamics in a wide class of life-cycle models is driven by a mapping from permanent earnings, net worth, and age into saving outcomes. Reduced-

[25] It is well known that kernel-based methods suffer from boundary bias. As observations get more disperse near the limits of the marginal distributions' support distant observations become disproportionately influential and bias the estimator towards the mean. An Epanechnikov kernel does not eliminate the bias but has compact support and hence uses no information from distant observations. Hence, given the shape of the quantiles, the non-monotonicity in the relationship is, if anything, more strongly pronounced than suggested by the graph.

form estimation of such mapping's empirical counterpart is informative about the aspects in which different frameworks are able to account for the data. We exploit the panel dimension of the data to construct proxies for permanent earnings and explicit measures of household saving based on observed changes of net worth across periods.

Given the panel's short duration, explicit total saving of individual households is observed only over a period of three or four years. It is plausible that households with given permanent earnings will have a much more variable distribution of annual as opposed to long-term saving outcomes. Given this and the likelihood of extreme observations we choose to model the median of the conditional distribution. Formally, we are interested in estimating

$$Median(A_{i,t+1} - A_{i,t}) = F(A_{i,t}, \hat{w}_i, age_{i,t}) \tag{15}$$

where $\hat{w}$ is appropriately defined measure of permanent earnings[26]. We confine attention to linear in parameters form for $F(.)$ but allow for the form of the linear dependence to differ across specifications (see below). Two key issues are apparent immediately. First, observed net worth at time t enters on both sides. As measurement error is likely in net worth records, direct estimation of (15) will identify severely downwards biased relationship (in addition to the standard measurement error problems for quantile regression). Second, permanent earnings are unobserved and have to be predicted at a first stage in a way consistent so that the second stage is estimated by a median regression.

To tackle measurement error in net worth we use lagged, rather than contemporaneous observations of net worth as regressors. In particular, the specifications for the 1996 panel use $A_{i,t-2}$ instead of $A_{i,t}$ as regressors, and the results from the 2001 panel use $A_{i,t-1}$ instead[28]. It should be noted that lagged net worth is not treated as in instrument for current net

[26]Recall that in the model saving is age-independent conditional on wealth. It should be noted that this is, first, inconsistent with standard versions of the life-cycle model and, second, obtains as retirement plans rather than age determine expected lifetime income. Including age[27] as a regressor in (15) allows for testing the relevance of this implication; furthermore, if age effects are actually important, accounting for them is desirable in modelling the relationship of interest.

[28]Recall, that the data contains four net worth observations for the 1996 panel and three for the 2001 panel.

worth. More precisely, consider two lags of the discrete time counterpart of (13)

$$\begin{aligned} A_{i,t+1} - A_{i,t} &= \frac{e^r - 1}{r}(rA_{i,t} + w_i - c(w_i)) \\ A_{i,t} - A_{i,t-1} &= \frac{e^r - 1}{r}(rA_{i,t-1} + w_i - c(w_i)) \end{aligned}$$

and substitute the solution for $A_{i,t}$ from the second equation into the first

$$A_{i,t+1} - A_{i,t} = \frac{e^r(e^r - 1)}{r}(rA_{i,t-1} + w_i - c(w_i))$$

The latter suggests that the relationship between $A_{i,t+1} - A_{i,t}$ and $rA_{i,t-1}$ conditional on permanent earnings is just scaling up the coefficients of the original relationship by proportion $e^r \approx (1 + r)$. As long as the measurement error in wealth is of the form $(\mathrm{me}_i + \mathrm{me}_{i,t})$ and $\mathrm{me}_{i,t}$ is not serially correlated, this eliminates the bias due to $A_{i,t}$ entering both sides of the equation.

In order to construct proxies for permanent earnings we follow the literature and instrument current earnings in a first stage including the instruments as well as the other second-stage regressors. We consider as instruments lagged labour income and education. Education is likely a suitable instrument as it does not vary considerably in the population of middle aged households and is strongly related to permanent labour income. In particular, we use the interaction of the education levels (grouped into four categories) of the household head and spouse. Lagged earnings are likely correlated with the permanent component of earnings, however, they also include lagged transitory shocks. It is intuitive that the longer the lag the more convincing the instrument is. Given the length of the SIPP panel the longest lag we can use is three years in the 1996 data and two years in the 2001 data (that is instrument earnings at $t+1$ with earnings at $t-2$ in the 1996 panel and $t-1$ in the 2001 panel) but this is likely sufficient for household-level data[29]. As an empirical justification for

[29] For example, Blundell *et al.* (2008) find no evidence of a moving average component in excess of MA(1) in income growth data from the PSID.

the suitability of the instruments, note that Dynan *et al.* (2004) find permanent income proxies based on measures of education, lagged earnings or consumption to imply very similar conclusions about the relationship between savings and permanent income in three different US surveys. Once the first-stage equation is estimated the permanent earnings proxies are constructed as the fitted-values at constant age.

To limit the influence of extreme earnings observations, the first-stage is also fitted by LAD. The whole two-stage procedure, originally proposed by Amemiya (1982), gives rise to the double-stage least absolute deviations (DSLAD) estimator. Following his suggestion, we redefine the dependent variable for the second stage as $p\Delta A_t + (1-p)\widehat{\Delta A_t}$, where $\Delta A_t \equiv A_{t+1} - A_t$, $\widehat{\Delta A_t}$ is the fitted value from a median regression of ΔA_t on all explanatory variables from the first stage, and $p \in [0,1]$ is a value chosen by the econometrician[30]. Consistently with Amemiya (1982) second-stage estimates are reported for two different values of p - 0.2 and 0.5 - but we also implement robustness checks using other values, including $p = 1$. In all results that follow we report bootstrapped standard errors based on 1000 pairwize replications.

5.2.1 Estimation results

Given the above discussion we start by estimating two separate first-stage LAD regressions - one to construct our proxies and another to obtain $\widehat{\Delta A_t}$. We estimate them separately to insure against potential correlation between the error terms. The results are reported in Table 6. Unsurprisingly, the instruments are highly relevant and there is a significant (although small in magnitude) age effect. We construct the permanent earnings proxy as the prediction from the first equation at constant age (the mean age in each sample) and split households into four quartiles based on this measure. To allow for non-linear relationship

[30]This reformulation of the dependent variable was suggested by Amemiya (1982) as a generalisation of a 2SLS property. While no formal procedure for choosing p was proposed, it was suggested that p in the range of 0.2 to 0.5 leads to an estimator that significantly outperforms $p = 1$ in terms of efficiency. Kim and Muller (2004) later demonstrated that the choice of p is inconsequential in median regressions except in very small samples.

we introduce permanent earnings in the second stage through a quartile indicator. Note that while the proxy likely suffers from measurement error, this has no consequences for the estimation as long as permanent-earnings quartiles are identified consistently.

As a baseline specification for the conditional median we estimate a second stage of observed wealth changes on permanent earnings quartile indicator, lagged net worth (included linearly) and age categories. Table 3 (4) reports the estimated coefficients for $p = 0.2$ ($p = 0.5$). Columns (1) to (5) ((6) to (10)) are estimated in the 1996 (2001) sample. The first specification (columns (1) and (6)) introduces permanent earnings through the proxy linearly, while the other columns allow for non-linearity by using quartile indicators. The second specification (columns (3) and (8)) allows for non-linear relationship with respect to net worth (including a square term) - the estimated coefficient is close to and insignificantly different from zero. Columns (4) and (9) further allow for an interaction of net worth and age, and yield estimates close to and insignificantly different from zero. The last specification (columns (5) and (10)) allows for interaction of net worth with earnings quartiles and yields coefficients insignificant from zero[31].

As a further illustration of the results Figure 9 plots the predicted median saving for the four earnings categories (obtained from columns (2) and (6) of Tables 3 4) against the median value of the proxy variable in each category, holding age and net worth constant. The profiles are increasing and approximately linear. Interpreted directly through the model, such earnings profiles are consistent with approximately constant saving rates out of permanent earnings.

To allow for further flexibility of the functional form we split the households of each permanent earning category into four quartiles of observed lagged net worth and estimate a second-stage median regression of total savings on the interaction of the identified permanent earnings and net worth quartiles, as well as age indicators. This relaxes the assumptions of linear wealth effect and separability between wealth and earnings on the right-hand side,

[31]For all specifications we include all second-stage regressors in the first stage. The respective first-stage coefficients are also found insignificant. Results are not reported for brevity.

Table 3: Second stage, DSLAD, $p = 0.2$

	1996					2001				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(Intercept)	79.853	8083.005	6508.685	8528.913	7197.487	6458.820	4318.103	3998.990	5648.287	2197.275
	(5453.868)	(5193.896)	(5315.960)	(5585.134)	(5509.939)	(6041.422)	(6324.514)	(6176.372)	(6606.533)	(6165.757)
NW_{t-1}	0.022***	0.026***	0.029***	0.040	0.021***	0.026***	0.032***	0.029***	0.056*	0.020·
	(0.004)	(0.004)	(0.004)	(0.025)	(0.006)	(0.004)	(0.004)	(0.005)	(0.026)	(0.011)
PE	0.183***					0.238***				
	(0.012)					(0.011)				
(25,50]		2206.355***	2130.518***	2203.090***	2055.511***		4600.603***	4645.316***	4606.590***	4012.096***
		(327.077)	(318.851)	(333.625)	(403.831)		(391.492)	(405.155)	(408.546)	(501.473)
(50,75]		5728.661***	5631.567***	5709.911***	5102.821***		8923.701***	9132.997***	8965.278***	9356.404***
		(416.074)	(429.798)	(455.473)	(580.847)		(546.149)	(570.297)	(583.121)	(920.485)
(75,100]		12479.740***	12257.491***	12433.661***	12452.838***		16134.717***	16344.759***	16351.213***	15698.244***
		(676.300)	(660.578)	(682.962)	(1057.155)		(1059.636)	(1011.142)	(1097.004)	(1260.122)
Age	-162.090	-264.150	-182.073	-296.260	-211.316	-435.180	-12.516	-6.892	-104.298	84.788
	(262.788)	(250.247)	(255.938)	(275.777)	(265.924)	(285.298)	(303.428)	(294.855)	(326.001)	(296.119)
Age^2	1.555	2.956	1.882	3.444	2.272	3.850	-1.318	-1.249	0.049	-2.281
	(3.106)	(2.972)	(3.028)	(3.351)	(3.152)	(3.263)	(3.540)	(3.428)	(3.920)	(3.461)
NW^2_{t-1}			-0.000					0.000		
			(0.000)					(0.000)		
$NW_{t-1} \times Age$				-0.000					-0.001	
				(0.001)					(0.001)	
$NW_{t-1} \times$ (25,50]					0.006					0.021
					(0.009)					(0.015)
$NW_{t-1} \times$ (50,75]					0.012					0.004
					(0.009)					(0.014)
$NW_{t-1} \times$ (75,100]					0.004					0.012
					(0.010)					(0.013)
Num. obs.	4946	4946	4946	4946	4946	4780	4780	4780	4780	4780

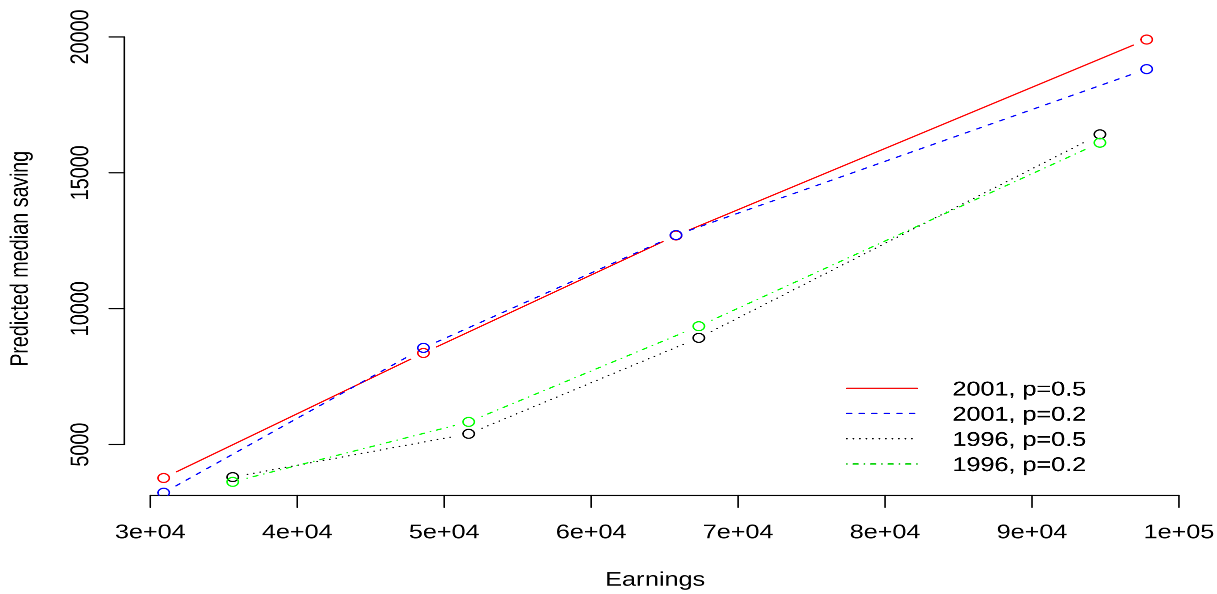
Bootstrap standard errors in parentheses. Controls omitted. NW: Net worth, PE: permanent earnings proxy.

Table 4: Second stage, DSLAD, $p = 0.5$

	1996					2001				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(Intercept)	-4316.045	3370.254	849.604	4436.982	-2800.177	-5425.998	338.977	-483.047	828.013	2913.194
	(9410.888)	(10583.272)	(10808.451)	(11022.162)	(11508.888)	(11012.092)	(11297.280)	(11145.651)	(11593.610)	(11022.863)
NW_{t-1}	0.021**	0.025**	0.035***	0.048	0.021·	0.027**	0.033**	0.028*	0.044	0.011
	(0.008)	(0.009)	(0.010)	(0.053)	(0.012)	(0.009)	(0.010)	(0.013)	(0.052)	(0.020)
PE	0.189***					0.236***				
	(0.023)					(0.029)				
(25,50]		1592.829*	1380.043·	1591.785*	1349.212		5330.609***	5288.238***	5195.955***	3226.520***
		(688.902)	(707.393)	(708.934)	(843.606)		(828.015)	(873.727)	(820.585)	(859.348)
(50,75]		5123.292***	4910.054***	5052.301***	4049.439**		9482.225***	9858.373***	9454.068***	10453.954***
		(846.812)	(862.645)	(902.518)	(1245.206)		(1294.843)	(1311.377)	(1265.005)	(1849.279)
(75,100]		12612.822***	12076.502***	12643.825***	13584.305***		15589.445***	16369.478***	15935.964***	15878.239***
		(1644.112)	(1531.269)	(1617.001)	(2261.109)		(2286.390)	(2154.795)	(2227.410)	(3036.421)
Age	32.235	-35.554	96.673	-111.280	272.692	79.476	97.273	141.641	61.679	4.577
	(452.162)	(506.304)	(516.757)	(543.025)	(555.280)	(529.083)	(532.290)	(525.117)	(567.303)	(524.174)
Age^2	-0.742	0.299	-1.517	1.454	-3.431	-1.684	-2.041	-2.544	-1.459	-1.110
	(5.419)	(5.940)	(6.076)	(6.575)	(6.560)	(6.195)	(6.135)	(6.050)	(6.848)	(6.107)
NW^2_{t-1}			-0.000					0.000		
			(0.000)					(0.000)		
$NW_{t-1} \times Age$				-0.000					-0.000	
				(0.001)					(0.001)	
$NW_{t-1} \times$(25,50]					0.010					0.052*
					(0.020)					(0.025)
$NW_{t-1} \times$(50,75]					0.023					0.010
					(0.022)					(0.028)
$NW_{t-1} \times$(75,100]					-0.001					0.020
					(0.020)					(0.024)
Num. obs.	4946	4946	4946	4946	4946	4780	4780	4780	4780	4780

Bootstrap standard errors in parentheses. Controls omitted. NW: Net worth, PE: permanent earnings proxy.

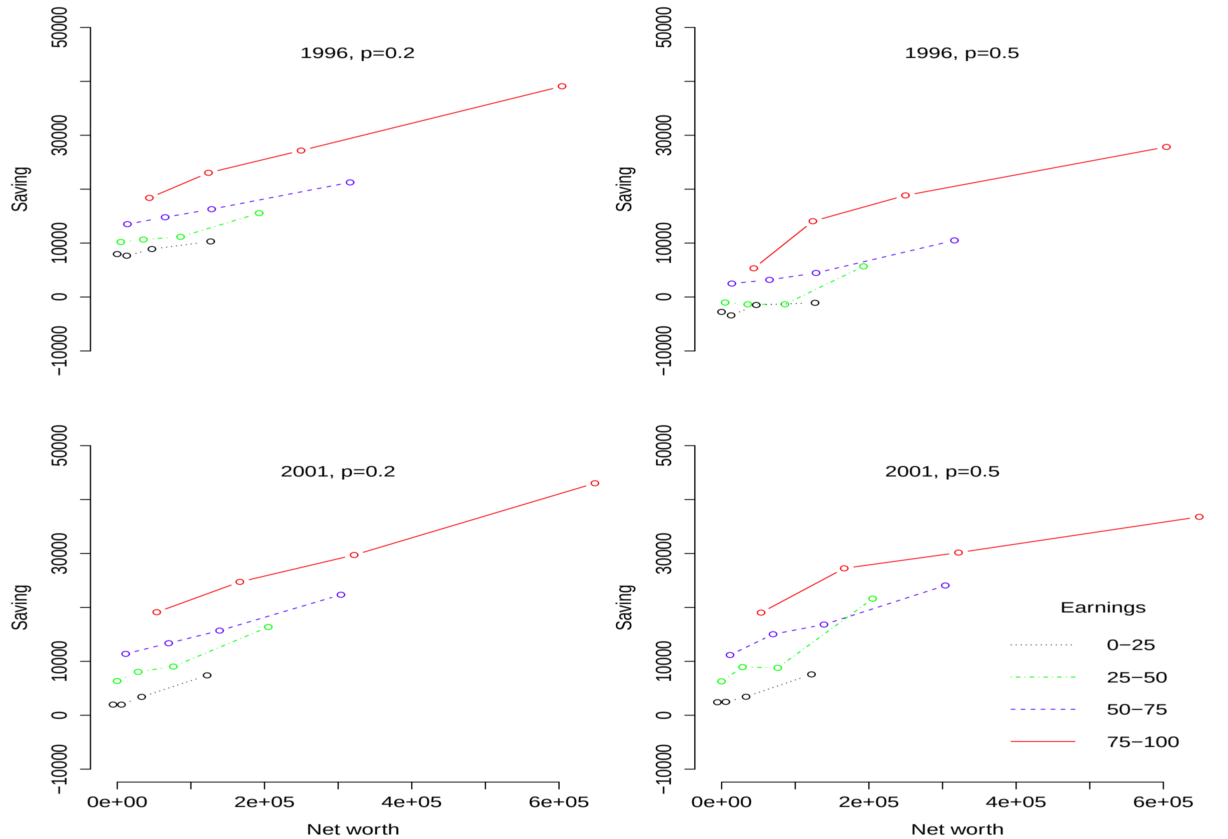
Figure 9: Earnings categories coefficients



hence allowing for much more general functional forms. The estimated second-stage coefficients (which identify median savings for the respective group of households) are reported in Table 7 and plotted in Figure 10 against the median lagged net worth of the respective groups of households. Inspection of results suggests that the extra flexibility allowed brings little extra insight. The wealth profiles are similar across permanent-earnings quartiles and approximately linear, reasserting the previous results.

In summary, median total saving increases substantially and significantly with permanent earnings. Saving is positively associated with net worth and the relationship is adequately approximated as linear; the coefficient of proportionality is within the magnitude of the real interest rate; and is approximately constant across age and permanent earnings groups. Households with high permanent earnings move quickly to the right of the net worth distribution and over time accumulate significantly larger stocks of wealth as compared to their less-earning counterparts. Conditional on permanent earnings and net worth, the differences in saving of households of different age are statistically insignificant. These results are strongly suggestive of the adequacy of (13) as describing household-level wealth dynamics.

Figure 10: Median saving by permanent earnings/wealth category, conditional



In particular, the model's implications for saving behaviour account for the observed saving outcomes in reduced form. Interestingly, the apparent age-invariance of saving and the increasing relationship with net worth (conditional on age and earnings) are not only supportive of the model, but also at odds with the implications of canonical life-cycle models[32]. While purely descriptive, the results are suggestive for the model's adequacy in explaining households' median saving outcomes and the evolution of wealth distributions in the presence of permanent earnings differentials and heterogeneity in initial wealth endowments.

[32]In standard life-cycle models a wealthy household saves less than a wealth poor household of the same permanent income and age. Further, the mapping from net worth and permanent income onto saving is age-dependent.

5.3 The distribution of wealth and earnings

We next turn attention to the age profile of the conditional-on-earnings net worth distribution. We split the sample households into three age groups - $[30, 40)$, $[40, 50)$, and $[50, 60)$ - and estimate non-parametrically the net worth distribution conditional on earnings. Figure 11 presents the associated contour maps. The basic pattern identified in Figure 8 is still present and a few additional observations emerge. First, the distribution of net worth gets increasingly dispersed with age. Second, the higher dispersion is driven by lengthening of the right tail while the 10th percentile is relatively stable. Third, the increase in dispersion is more pronounced at high earnings. Fourth, the 10th conditional percentile is stable for low-earnings households and only increases with age at high earnings.

This pattern is qualitatively consistent with the model's predictions (Section 3) as long as preference parameters imply positive relationship between savings and earnings over the empirical support of the distribution. In particular, the "absorbing" property of low wealth is consistent with the data and the level of wealth where savings average zero is negatively related to earnings. To give some concrete substance to this claim we conduct the following exercise. First, we obtain the 10th, 50th and 90th percentile of wealth for households aged $[40, 50)$ and $[50, 60)$ in the four earnings quartiles. Then, using the observed earnings and net worth of households aged $[30, 40)$, we use the model to simulate their net worth 10 and 20 years into the future for different values of u_b. The simulations assume that earnings remain constant over time and that preferences are given by a log-utility function. We choose the value ($u_b = 1.029$) that minimizes the sum of absolute deviations between simulated and observed median net-worth levels of the four earnings quartiles in the 2001 sample by equally weighting each of the resulting 8 targets. Table 5 reports the observed and simulated quantiles of net worth by earnings and age group, using the same parameterisation in the 1996 sample. Further, as before we estimate non-parametrically the conditional distribution of net worth on the simulated data and present the associated contour plots in Figure 12.

Before interpreting the results a discussion is in order. First, this procedure makes no

Figure 11: Conditional distribution of net worth on earnings and age, data

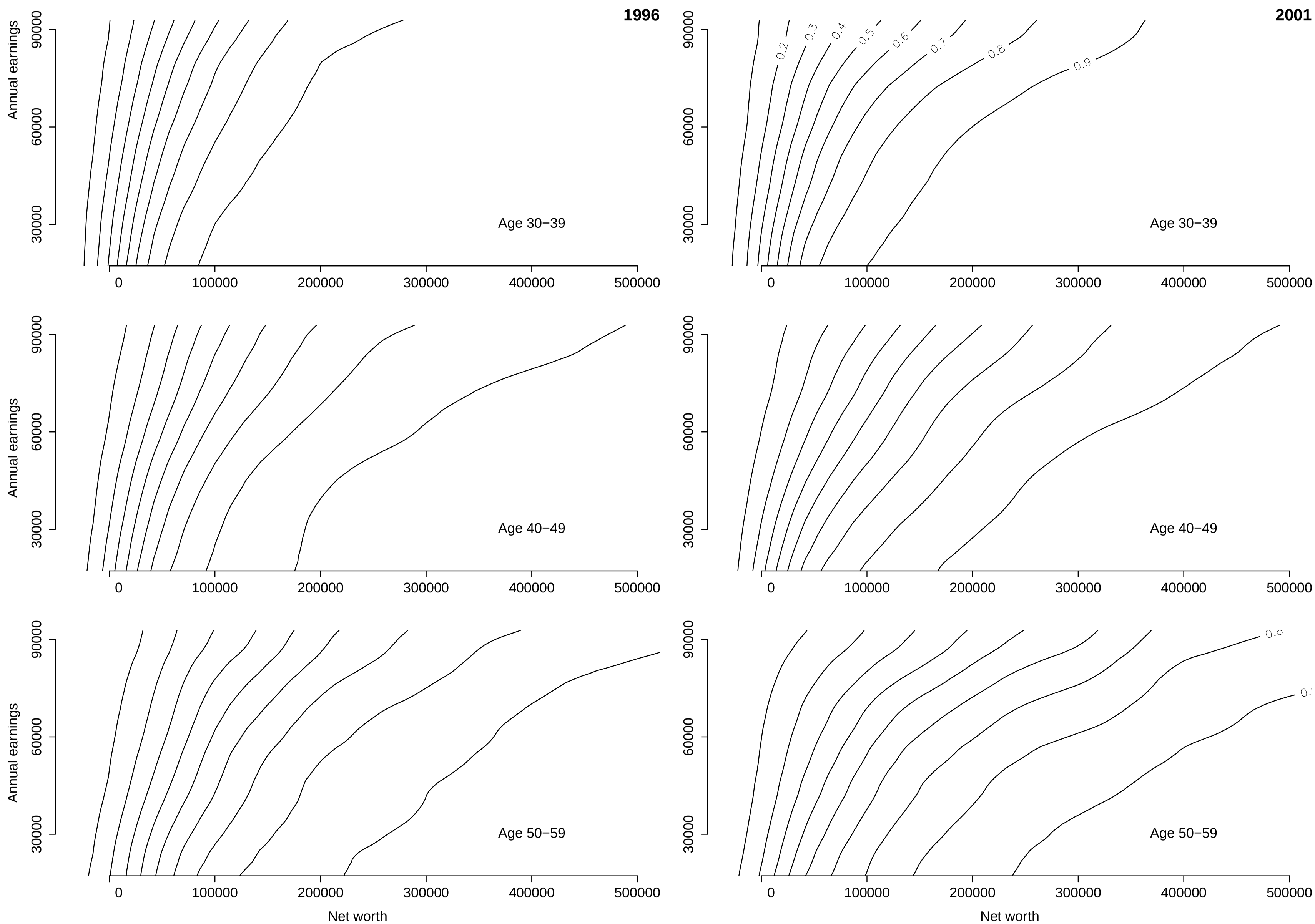


Figure 12: Conditional distribution of net worth on earnings and age, model, $\eta = 1$, $u_b = 1.029$

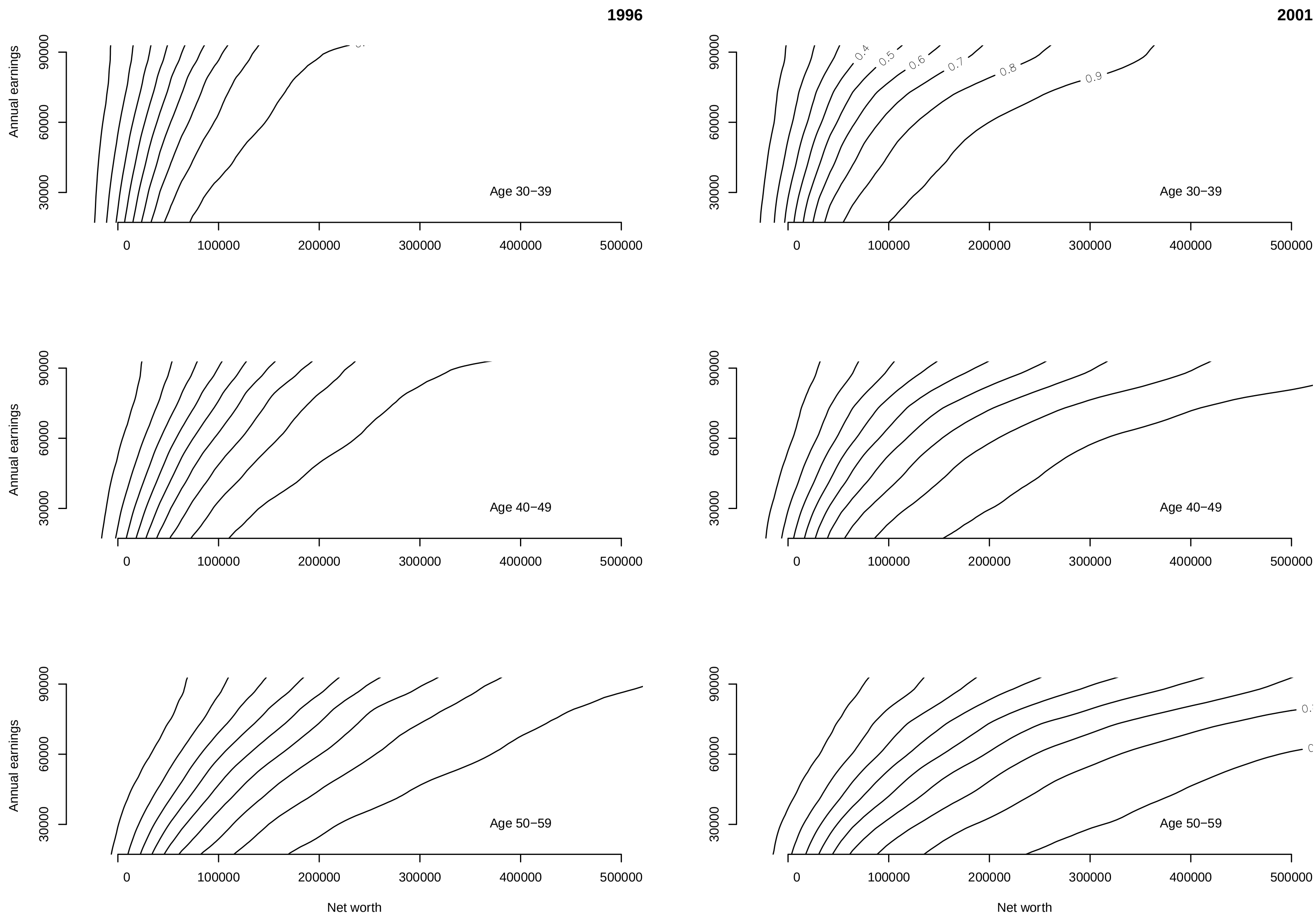


Table 5: Observed and fitted wealth quantiles

2001								
	Data				Model			
	E1	E2	E3	E4	E1	E2	E3	E4
	Age 40-49							
Q10	-6841	2827	15247	49527	-7537	490	12817	41715
Q25	2167	20528	51319	126931	6075	19476	41980	104865
Q50	22477	67590	113063	232817	18979	51134	89908	223971
Q75	76266	158469	206203	410325	66971	117125	180800	434227
Q90	168246	260598	394196	703753	154617	224718	299780	686177
	Age 50-59							
Q10	-4889	2625	10582	91449	-5422	15411	38913	88305
Q25	6078	33789	66748	178388	14313	43135	83978	187528
Q50	42096	91965	157163	330204	36894	90350	156483	371581
Q75	118628	202918	330131	596526	109036	190570	289049	685259
Q90	233016	372957	468283	979331	241446	350457	467381	1062598
1996								
	Data				Model			
	E1	E2	E3	E4	E1	E2	E3	E4
	Age 40-49							
Q10	-3182	912	5436	25902	-724	931	15424	30440
Q25	1947	15808	36059	62030	8815	23007	35035	69435
Q50	23211	46145	76251	139241	23302	47526	67208	130543
Q75	69531	97289	150125	275295	62076	96854	132254	223013
Q90	151776	174502	248018	512001	103201	164046	218501	448159
	Age 50-59							
Q10	-1843	11732	25291	42452	7443	14096	42565	74693
Q25	11660	37942	58883	93251	21370	48390	70532	130653
Q50	49798	90335	128139	202956	42722	86969	120161	224257
Q75	110797	157556	238351	400601	101073	158677	217278	365676
Q90	231892	301952	398360	666746	160582	259395	345046	713299

difference between permanent and transitory earnings components - observed earnings are treated as a proxy for long term earnings. Second, it abstracts from possible heterogeneity in preference parameters in the population. Third, the model is extremely parsimonious - the above exercise amounts to fitting 8 targets by varying a single parameter. Fourth, as reflected in Table 5 the procedure systematically underestimates the wealth accumulation over the first ten years, with an offsetting error over the next ten-year period. Subject to this, the simulated wealth profiles fit their empirical counterparts very well and the simulated conditional quantiles in Figure 12 are very close to their observed counterparts in Figure

11. It should be noted that by considering quantiles rather than moments we make no attempt to describe extreme wealth observations - an issue that has attracted considerable attention in the literature and that dominates measures of inequality based on statistics such as the Gini index. There are, however, good reasons to think that extreme wealth accumulation is not best understood as a life-cyclical phenomenon[33]. The results suggest that wealth dynamics could be adequately attributed to pure life-cycle motives when retirement is efficient; wealth inequality reflects earnings inequality to a significant extent; while highly stylized, the theoretical description of wealth dynamics in Section 3 provides an adequate approximation to the data.

6 Conclusion

This paper studies the labour-market decisions of risk-averse workers in a world of frictional labour markets. The existence of labour-market rigidities imply that the tradeoff between consumption and leisure is resolved over the life cycle, with individuals working while young and saving in order to retire later. The model has highly tractable implications for wealth dynamics and educational investment which emphasise pure life-cycle motives, labour-market decisions (including optimal retirement), persistent earnings differentials and heterogeneity in initial wealth endowments. Using data from the Survey of Income and Program Participation we document how the household-level distribution of wealth and earnings evolves with age and how permanent earnings and wealth associate with median saving outcomes. The theoretical framework provides clear interpretation for the observed regularities. The evidence is suggestive that life-cycle motives and permanent-earnings heterogeneity are accountable for a significant part of the between-household differences in wealth.

[33] See Carroll (2000) for a discussion.

References

Alan, S., Atalay, K. and Crossley, T. F. (2015). Do the Rich Save More? Evidence from Canada. *Review of Income and Wealth*, **61** (4), 739–758.

Amemiya, T. (1982). Two Stage Least Absolute Deviations Estimators. *Econometrica*, **50** (3), 689–711.

Benartzi, S. and Thaler, R. H. (2013). Behavioral Economics and the Retirement Savings Crisis. *Science Magazine (New York, N.Y.)*, **339** (March), 1152–3.

Bender, K. A., Mavromaras, K., Theodossiou, I. and Wei, Z. (2014). The Effect of Wealth and Earned Income on the Decision to Retire: A Dynamic Probit Examination of Retirement. *IZA Discussion Paper Series*, (7927).

Bernheim, B. D., Skinner, J. and Weinberg, S. (2001). What accounts for the variation in retirement wealth among U.S. households? *American Economic Review*, **91** (4), 832–857.

Bloom, D. E., Canning, D., Mansfield, R. K. and Michael Moore (2007). Demographic change, social security systems, and savings. *Journal of Monetary Economics*, **54** (1), 92–114.

—, — and Moore, M. (2014). Optimal Retirement with Increasing Longevity. *Scandinavian Journal of Economics*, **116** (3), 838–858.

Blundell, R., Pistaferri, L. and Preston, I. (2008). Consumption inequality and partial insurance. *American Economic Review*, **98** (5), 1887–1891.

Brown, J. R., Coile, C. C. and Weisbenner, S. J. (2010). The Effect of Inheritance Receipt on Retirement. *Review of Economics and Statistics*, **92** (2), 425–434.

Burbidge, J. B. and Robb, A. L. (1980). Pensions and Retirement Behaviour. *Canadian Journal of Economics*, **13** (3), 421–437.

Cagetti, M. (2003). Wealth Accumulation Over the Life Cycle and Precautionary Savings. *Journal of Business & Economic Statistics*, **21** (3), 339–353.

Carroll, C. D. (2000). Why Do the Rich Save So Much? In J. B. Slemrod (ed.), *Does Atlas Shrug? The Economic Consequences of Taxing the Rich.*, *14*, Harvard University Press, pp. 465–84.

— and Samwick, A. A. (1997). The Nature of Precautionary Wealth. *Journal of Monetary Economics*, **40** (1), 41–71.

Czajka, J. L., Jacobson, J. E. and Cody, S. (2003). Survey Estimates of Wealth: A Comparative Analysis and Review of the Survey of Income and Program Participation. *Social Security Bulletin*, **65** (1), 63–69.

D'Albis, H., Lau, S. H. P. and Sánchez-Romero, M. (2012). Mortality transition and differential incentives for early retirement. *Journal of Economic Theory*, **147** (1), 261–283.

Dynan, K. E., Skinner, J. and Zeldes, S. P. (2004). Do the Rich Save More? *Journal of Political Economy*, **112** (2), 397–444.

French, E. (2005). The Effects of Health, Wealth, and Wages on Labour Supply and Retirement Behavior. *Review of Economic Studies*, **72** (2), 395–427.

Fulford, S. L. (2015). The surprisingly low importance of income uncertainty for precaution. *European Economic Review*, **79**, 151–171.

Gourinchas, P.-O. and Parker, J. A. (2002). Consumption over the Life Cycle. *Econometrica*, **70** (1), 47–89.

Guiso, L., Jappelli, T. and Terlizzese, D. (1992). Earnings uncertainty and precautionary saving. *Journal of Monetary Economics*, **30** (2), 307–337.

Hubbard, R. G., Skinner, J. and Zeldes, S. P. (1994a). Expanding the Life-Cycle Model - Precautionary Saving and Public Policy. *American Economic Review*, **84** (2), 174–179.

—, — and — (1994b). The importance of precautionary motives in explaining individual and aggregate saving. *Carnegie-Rochester Confer. Series on Public Policy*, **40** (C), 59–125.

Huggett, M. and Ventura, G. (2000). Understanding why high income households save more than low income households. *Journal of Monetary Economics*, **45**, 361–397.

Hurst, E., Lusardi, A., Kennickell, A. and Torralba, F. (2010). The Importance of Business Owners in Assessing the Size of Precautionary Savings. *Review of Economics and Statistics*, **92** (1), 61–69.

Imbens, G. W., Rubin, D. B. and Sacerdote, B. I. (2001). Estimating the Effect of Unearned Income on Labor Earnings , Savings , and Consumption : Evidence from a Survey of Lottery Players. *American Economic Review*, **91** (4), 778–794.

Kallestrup-Lamb, M., Kock, A. B. and Kristensen, J. T. (2016). Lassoing the Determinants of Retirement. *Econometric Reviews*, **35** (8-10), 1522–1561.

Kim, T.-H. and Muller, C. C. (2004). Two-stage quantile regression when the first stage is based on quantile regression. *Econometrics Journal*, **7**, 218–231.

Kuhn, M., Wrzaczek, S., Prskawetz, A. and Feichtinger, G. (2015). Optimal choice of health and retirement in a life-cycle model. *Journal of Economic Theory*, **158**, 186–212.

Laibson, D. I., Repetto, a. and Tobacman, J. (1998). Self-control and saving for retirement. *Brookings Papers on Economic Activity*, **1** (1), 91–172.

Li, Q. and Racine, J. S. (2008). Nonparametric Estimation of Conditional CDF and Quantile Functions With Mixed Categorical and Continuous Data. *Journal of Business & Economic Statistics*, **26** (4), 423–434.

Lusardi, A. (1998). On the Importance of the Precautionary Saving Motive. *American Economic Review*, **88** (2), 449–453.

Rogerson, R. and Wallenius, J. (2013). Nonconvexities, retirement, and the elasticity of labor supply. *American Economic Review*, **103** (4), 1445–1462.

Schunk, D. (2009). What determines household saving behavior? an examination of saving motives and saving decisions. *Journal of Economics and Statistics (Jahrbuecher fuer Nationaloekonomie und Statistik)*, **229** (4), 467–491.

Toossi, M. (2013). Labor force projections to 2022: the labor force participation rate continues to fall. *Monthly Labor Review*, (December), 1–28.

Appendices

A Proofs and derivations

A.1 Derivation of Bellman equations

Let Δ be a discrete interval of time and $V(A,w) \equiv \max\{V_e(A,w), V^n(A,w)\}$.

Consider an employed worker with $\{A,w\}$. Her discrete-time Bellman equation is (suppressing the second argument for brevity)

$$V^e(A) = \max_c \{u(c)\Delta + \frac{1}{1+r\Delta} V(A')\}$$

where

$$A' = (1+r\Delta)A + w\Delta - c\Delta$$

Multiplying both sides by $(1+r\Delta)$ and subtracting $V(A,w)$

$$rV^e(A) = \max_c \{u(c) + \frac{V(A') - V(A)}{\Delta} + ru(c)\Delta\}$$

In the limit as Δ approaches 0 this implies (2).

Consider a worker with $\{A,w\}$ such that $V(A,w) = V^n(A,w)$. Her discrete-time Bellman equation is

$$V^n(A) = \max \left\{ \begin{array}{l} \max_{c\geq 0}[u(c)\Delta + u_b\Delta + \frac{1}{1+r\Delta} V^n(A'_{np})] \\ \\ \max_{c\geq 0}[u(c)\Delta + \frac{1}{1+r\Delta}(\lambda\Delta \max(V^n(A'_{js}), V^e(A'_{js})) + \\ \\ \qquad\qquad\qquad +(1-\lambda\Delta)V^n(A'_{js}))] \end{array} \right\}$$

where the maximization is conditional on

$$\begin{aligned} A'_{np} &= (1+r\Delta)A - c\Delta \\ A'_{js} &= (1+r\Delta)A + b\Delta - c\Delta \end{aligned} \tag{16}$$

Multiplying both sides by $(1+r\Delta)$ and subtracting $V(A,w)$

$$rV^n(A) = \max \left\{ \begin{array}{l} \max_{c\geq 0}[u(c) + u_b + \frac{V^n(A'_{np}) - V^n(A)}{\Delta}] \\ \\ \max_{c\geq 0}[u(c) + \frac{V^n(A'_{js}) - V^n(A)}{\Delta} + \lambda \max(V^e(A'_{js}) - V^n(A'_{js}), 0)] \end{array} \right\}$$

In the limit as Δ approaches 0 this implies (1).

A.2 Proof of Proposition 1

Under property 1 the Bellman equation for an employed worker can be stated as

$$rV^e(A,w) = \max \left\{ \begin{array}{l} \max_{c\geq 0}[u(c) + \frac{\partial V^e(A,w)}{\partial A}(rA + w - c)] \\ u(rA) + u_b \end{array} \right.$$

Under the conjecture

$$rV(A,w) = \left\{ \begin{array}{ll} u(rA+w) & \text{if } A \leq \underline{A}^E \\ u(c^*) + (rA + w - c^*)u'(c^*) & \text{if } A \in (\underline{A}^E, \bar{A}^E) \\ u(rA) + u_b & \text{if } A \geq \bar{A}^E \end{array} \right.$$

Suppose $A \leq \underline{A}^E$. The Bellman equation of the worker is then

$$rV^e(A,w) = \max\{\max_c[u(c) + u'(rA+w)(rA+w-c)], u(rA) + u_b\}$$

Maximization implies that optimal consumption is $c^e = rA + w$. Substituting above

$$rV^e(A, w) = \max \left\{ \begin{array}{c} u(rA + w) \\ u(rA) + u_b \end{array} \right\}$$

The payoff from permanent retirement is not defined for negative wealth, hence working forever dominates if $\underline{A}^E \leq 0$ or if $A < 0 < \underline{A}^E$. Suppose $0 < A < \underline{A}^E$. Then both payoff functions increase in A and the payoff from retirement increases faster. At $A = \underline{A}^E$, $u(rA + w) = u(c^*)$ and $u(rA) + u_b = u(c^* - w) + u_b = u(c^* - w) + wu'(c^*)$. As $u(.)$ is convex, $u(c^*) - u(c^* - w) - wu'(c^*) > 0$. Therefore, working forever dominates and $rV^e(A, w) = u(rA + w)$ as conjectured.

Consider $A \in (\underline{A}^E, \bar{A}^E)$. The Bellman equation under the conjecture is

$$rV^e(A, w) = \max \left\{ \begin{array}{cc} \max_c & [u(c) + u'(c^*)(rA + w - c)] \\ u(rA) + u_b & \end{array} \right\}$$

Optimal consumption is $c = c^*$. Substituting above

$$rV^e(A, w) = \max \left\{ \begin{array}{c} u(c^*) + u'(c^*)(rA + w - c^*) \\ u(rA) + u_b \end{array} \right\}$$

At the upper bound of the subset $u(c^*) + u'(c^*)(r\bar{A}^E + w - c^*) = u(c^*) + u_b = u((r\bar{A}^E) + u_b$, while the derivatives of the payoffs with respect to A satisfy $ru'(c^*) < ru'(rA)$ within the interior of the set. Hence saving for retirement dominates and $rV^e(A, w) = u(c^*) + u'(c^*)(rA + wic^*)$ as conjectured.

Finally, suppose $A \geq \bar{A}^E$. Under the conjecture the Bellman equation is

$$rV^e(A, w) = \max \left\{ \begin{array}{cc} \max_c & [u(c) + u'(rA)(rA + w - c)] \\ u(rA) + u_b & \end{array} \right\}$$

Optimal consumption is $c = rA$. Substituting above

$$rV(A,w) = \max \left\{ \begin{array}{c} u(rA) + u'(rA)w \\ u(rA) + u_b \end{array} \right\}$$

As $rA > c^*$ not working is strictly preferred and the conjecture is verified.

This completes the proof of Proposition 1.

A.3 Characterisation of V^n and solution

Consider a non-employed worker with $\{A, w\}$. As she freely changes state between non-participation and job-search, (1) and property 1 suggest that the worker optimally seeks employment when $bV^n_A(A,w) + \lambda(V^e(A,w) - V^n(A,w)) \geq u_b$ and does not participate otherwise. Let

$$S(A,w) \equiv bu'(c^n(A,w)) + \lambda(V^e(A,w) - V^n(A,w)) \tag{17}$$

Consider a non-employed worker at the natural borrowing limit, $A = -b/r$. It is immediate that the only behaviour not violating the limit is to search for a job and consume nothing. The Inada condition implies that $S(-b/r, w)$ tends to infinity. Let $\underline{A}^U > -b/r$ be an amount of wealth such that $S(A,w) > u_b, \forall A \in [-b/r, \underline{A}^U)$, i.e. job search dominates for all amounts of wealth below $\underline{A}^U$. Then for any $A \in [-b/r, \underline{A}^U)$ the Bellman equation (1) reduces to

$$(r+\lambda)V^n(A) = u(c^n(A)) + u'(c^n(A))(rA + b - c^n(A)) + \lambda V^e(A) \tag{18}$$

Total differentiation of (18) with respect to time yields

$$[u''(c^n(A))\dot{c^n}(A) + \lambda(V^e_A(A) - V^n_A(A))]\dot{A} = 0 \tag{19}$$

One possibility for optimal wealth and consumption dynamics is to consume all income, implying $c^n(A, w) = rA + b$ and $\dot{A} = 0$. It is later verified that this is never optimal in the interior of $[-b/r, \underline{A}^U)$. Alternatively, consider the set of strategies implying $\dot{A} < 0$. Then by (19)

$$\dot{c^n}(A) = \frac{\lambda(u'(c^n(A)) - u'(c^e(A))}{u''(c^n(A))} \tag{20}$$

Over the duration of a job search spell consumption is declining over time.

Lemma 1. ***Optimality of job search***

There exists a unique $\underline{A}^U < \bar{A}^E$ such that job search is strictly preferred to non-participation for all $A \in [-b/r, \underline{A}^U)$ and $S(\underline{A}^U, .) = bu'(c^n(\underline{A}^U, .)) + \lambda(V^e(\underline{A}^U, .) - V^n(\underline{A}^U, .)) = u_b$.

Proof. Differentiation of (17) with respect to A implies

$$S_A(A) = bu''(c^n(A, w))c^n_A(A) + \lambda(u'(c^e(A)) - u'(c^n(A))) < 0$$

and as already established job search is strictly preferred to non-participation at the borrowing limit:

$$\lim_{A \to -b/r} S(A) > u_b$$

In the limit as A tends to $\bar{A}^E$ property 1 and proposition 1 imply that non-participation is strictly preferred to job search. Therefore there exists a unique $\underline{A}^U$ such that $S(A) > u_b, \forall A \in [-b/r, \underline{A}^U)$ and $S(\underline{A}^U) = bu'(c^n(\underline{A}^U)) + \lambda(V^e(\underline{A}^U) - V^n(\underline{A}^U)) = u_b$. □

The dynamics of consumption of unemployed workers over the interval $A \in [-b/r, \underline{A}^U)$ is illustrated in Figure 2. The set of $\{A, c^n\}$ pairs where wealth is stationary are identified by the locus $0 = rA + b - c^n(A)$, represented by a straight line with slope r in the (A, c)-plane, with $c^n(-b/r) = 0$. Points below the line are consistent with wealth accumulation and points above imply decumulation. On the other hand, equation (20), suggests that consumption is in steady state when $c^n(A,.) = c^e(A)$, increases above the locus, and decreases below. The stable saddle path consistent with the terminal condition $c^n(-b/r) = 0$ therefore lies between the $\dot{A} = 0$ and $\dot{c}^n = 0$ loci and prescribes decreasing consumption and wealth during job search. The identified saddle path illustrated, however, only applies within the interval $A \in [-b/r, \underline{A}^U)$.

As $0 < b < w$, there exists a unique level of A, henceforth denoted A^{SS}, where both consumption and wealth are at steady state, and furthermore $A^{SS} \in (\underline{A}_E, \bar{A}^E)$. It is straightforward to verify that $A^{SS} = (c^* - b)/r$, $\underline{A}^U < A^{SS}$ and indeed optimal saving behaviour during job-search involves wealth decumulation[34].

Lemma 1 suggests that to the right of $\underline{A}^U$ there exists a neighbourhood, $A^{NP \geq JS}$, where non-participation weakly dominates job search. Differentiation of (17) with respect to A and evaluation at any point in $A^{NP \geq JS}$ implies

$$\frac{\partial S(A|A \in A^{NP \geq JS})}{\partial A} = bu''(c^n(A))\frac{\partial c^n(A)}{\partial A} + \lambda(u'(c^e(A) - u'(c^n(A))) < 0$$

under the assumptions (verified below) that consumption of non-employed is lower than that for employed and non-decreasing with wealth. Then non-participation is strictly preferred to job search for all $A > \underline{A}^U$.

Consider a non-employed worker with $A > \underline{A}^U$. As she optimally chooses non-participation, her value function reduces to

$$rV^n(A) = u(c^n(A)) + u_b + u'(c^n(A))(rA - c^n(A))$$

Totally differentiating with respect to time

$$u''(c^n(A))\dot{c}^n(A)(rA - c^n(A)) = 0 \tag{21}$$

[34]Is job search still optimal at A^{SS}? Suppose so. Then $S(A^{SS}) = u'(c^*)(\frac{br+\lambda w}{r+\lambda}) = u_b(\frac{br+\lambda w}{wr+w\lambda}) < u_b$ which is a contradiction.

identifying two potentially optimal consumption/saving strategies. One possibility is to consume all income, $c^n(A) = rA$ (Strategy i). As this implies $\dot{A} = 0$, if ever optimal this is optimal forever. Substituting into the value function the discounted lifetime payoff is

$$rV^n(A|i) = u(rA) + u_b. \tag{22}$$

Alternatively, (21) holds if $c^n_A(A) = 0$ implying $c^n(A) = c^n(\underline{A}^U)$ and wealth decumulates[35]. The discounted lifetime payoff is

$$\begin{aligned} rV^n(A|ii) &= u(c^n(\underline{A}^U)) + u_b + u'(c^n(\underline{A}^U))(rA - c^n(\underline{A}^U)) \\ &= rV^n(\underline{A}^U) + r(A - \underline{A}^U)u'(c^n(\underline{A}^U)) \end{aligned} \tag{23}$$

Let $\bar{A}^U \equiv c^n(\underline{A}^U)/r$. Note that Strategy ii) is only feasible when $A \leq \bar{A}^U$ as consuming $c^n(\underline{A}^U)$ implies wealth accumulation for higher A.

Lemma 2. ***Optimality of non-participation***

A non-employed worker optimally chooses to not participate and decumulate wealth if and only if $A \in [\underline{A}^U, \bar{A}^U]$. *Her value is function described by* (23).

A non-employed worker optimally chooses to not participate and consume all income if and only if $A > \bar{A}^U$. *Her value function is described by* (22).

Proof. Let

$$T(A) \equiv rV^n(A|ii) - rV^n(A|i) = u(c^n(\underline{A}^U)) - u(rA) + u'(c^n(\underline{A}^U))(rA - c^n(\underline{A}^U))$$

Evaluating at $A = \underline{A}^U$ and rearranging

$$\frac{T(\underline{A}^U)}{c^n(\underline{A}^U) - r\underline{A}^U} = \frac{u(c^n(\underline{A}^U)) - u(r\underline{A}^U)}{c^n(\underline{A}^U) - r\underline{A}^U} - u'(c^n(\underline{A}^U)) > 0$$

[35] Another possible strategy consistent with (21) is identified by $c^n_A(A) = 0$ and $\dot{A} > 0$. Such strategies are never optimal as worker never changes state yet does not maximize consumption but instead accumulates wealth indefinitely.

where the inequality follows from the fact that $c^n(\underline{A}^U) > r\underline{A}^U$ (recall figure 2) and concavity of $u(.)$.

Differentiation of $T(.)$ with respect to A yields

$$\frac{\partial T(A)}{\partial A} = r(u'(c^n(\underline{A}^U) - u'(rA)) \leq 0, \forall A \in [\underline{A}^U, \bar{A}^U]$$

with equality at $\bar{A}^U$. The inequality follows by the definition of $\bar{A}^U$.

Evaluating $T(.)$ at $A = \bar{A}^U$ yields $T(\bar{A}^U) = 0$. Therefore $T(A) \geq 0, \forall A \in [\underline{A}^U, \bar{A}^U]$ and strategy ii is optimal within this interval. Furthermore, strategy ii is infeasible for $A > \bar{A}^U$ implying the optimality of strategy i. This completes the proof of Lemma 2.

□

The analysis in this section fully characterises the solution of a non-employed worker's problem conditional on Property 1 and implies Proposition 2.

A.4 Proof of Theorem 1

Conditional on Property 1 the strategies described by Propositions 1 and 2 solve the Bellman equations (1) and (2) by construction. Next, we verify that the conjectured solution implies Property 1 as well.

Sufficient condition for Property 1 is $V^e(A) > V^n(A), \forall A < \bar{A}^E$. Since $c^* > c^n(\underline{A}^U)$ (Figure 2), $\bar{A}^E > \bar{A}^U$. Further, $V^e(\bar{A}^E) = V^n(\bar{A}^E)$ by Proposition 1.

Since $c^n(A) < c^e(A), \forall A < \bar{A}^E$ (Figure 3), it follows that $u'(c^e(A)) < u'(c^n(A))$ and equivalently $V^e_A(A) < V^n_A(A), \forall A < \bar{A}^E$.

Hence both $V^e(A)$ and $V^n(A)$ are increasing with A and $V^n(A)$ is increasing at a higher rate over $A < \bar{A}^E$. Therefore, $V^e(A) > V^n(A), \forall A < \bar{A}^E$ and Property 1 holds.

B Education efficiency frontier

For brevity let ”forever-workers”, ”savers”, and ”retirees” describe workers whose optimal strategy is to work forever, save for retirement and retire. The derivation of the education efficiency frontier proceeds by analysing the choice of education of workers based on their optimal employment strategies conditional on investing in education or not.

B.1 Forever-workers without education

Consider the set of workers whose optimal strategy would be to work forever if they chose to obtain no education. They are identified by the set of abilities and wealth endowments

$$FW_0 \equiv \{A_0, w_0 | rA_0 \leq c_0^* - w_0\}$$

Should they invest in education instead their optimal strategy might be to either work forever or to save for retirement[36].

B.1.1 Forever-workers with education

The subset

$$FW_0^{FW} \equiv \{A_0, w_0 | rA_0 \leq \min\{c_0^* - w_0, c_1^* - w_0(1+e) + rk\}\}$$

identifies forever-workers irrespective of education choice. They never enjoy leisure so their decision is driven solely by comparison of consumption possibilities with and without education. The latter depends both on wealth and earnings which are perfectly substitutable, and the benefit from extra earnings, $w_0 e$, is enjoyed in perpetuity. A worker optimally invests in education if and only if the perpetuity-discounted value of the extra flow of earnings exceeds

[36] Investment in education involves a depletion of wealth and an increase in c^*, consumption during the wealth accumulation phase. Therefore a forever worker or saver without education is never an immediate retiree with education.

the cost of education

$$\frac{w_0 e}{r} \geq k \tag{24}$$

Conditional on belonging to FW_0^{FW} optimality of education is independent of the wealth endowment. Workers with sufficiently high ability $w_0 > rk/e$ always pursue a degree. Less able workers, do not. The strict-equality counterpart of (24) identifies the education efficiency frontier over FW_0^{FW}.

B.1.2 Savers with education

The relative complement of FW_0^{FW} in FW_0

$$FW_0^S \equiv \{A_0, w_0 | rA_0 \in (c_1^* - w_0(1+e) + rk, c_0^* - w_0]\}$$

identifies forever-workers without education who save for retirement if they had education. The set is non-empty as long as $c_1^* - c_0^* < w_0 e - rk$ for some w_0[37]. Since $c^*(w)$ is increasing in w existence requires $w_0 e - rk > 0$, i.e. all members of FW_0^S lie above the education efficiency frontier for FW_0^{FW}. The value with education always exceeds the value without as

$$u(c_1^*) + \left(\frac{rA_1 + w_1 - c_1^*}{w_1}\right) u_b > u(c_1^*) > u(c_0^*) \geq u(rA_0 + w_0)$$

With education these workers achieve both higher consumption and the prospect of enjoying leisure some time in the future hence always invest.

In summary, workers who would work forever without education, optimally choose to pursue a degree if and only if $w_0 \geq rk/e$. Subject to this, their wealth endowment is irrelevant.

[37]The existence of this and some subsequently discussed sets depends on the form of preferences (see (10)). Figure 6 illustrates just one special case. For example, if risk aversion is low and the worker has weak preference for leisure, FW_0^S may be empty.

B.2 Savers without education

Consider the set of workers whose optimal strategy would be to save for retirement if they chose to obtain no education. They are identified by the set

$$S_0 \equiv \{A_0, w_0 | rA_0 \in (c_0^* - w_0, c_0^*)\}$$

Should they invest in education instead their optimal strategy might be to either work forever or save for retirement.

B.2.1 Forever-workers with education

The subset

$$S_0^{FW} \equiv \{A_0, w_0 | rA_0 \in (c_0^* - w_0, \min\{c_0^*, c_1^* - w_0(1+e) + rk\})\}$$

identifies the workers who would work forever were they to invest in education. To trade the perspective of future leisure they need sufficient increases in consumption.

S_0^{FW} is never empty. Consider a worker with $w_0 = rk/e$ and $rA_0 = c_0^*(rk/e) - rk/e$. This worker belongs to FW_0 and is just indifferent between working forever and saving for retirement, as well as between investing or not investing in education. In a neighbourhood to the right of $(rk/e, c_0^*(rk/e) - rk/e)$ (Figure 6), workers belong to S_0^{FW} (see discussion in section B.1.2).

Workers are indifferent to education if and only if

$$u(rA_0 - rk + w_0(1+e)) - u(c_0^*) = \frac{(rA_0 + w_0 - c_0^*)}{w_0} u_b \tag{25}$$

that is if the gain in consumption just equals the discounted value of leisure they forego.

Totally differentiating (25) with respect to A_0, the slope of the EEF is

$$\left.\frac{\partial w_0}{\partial A_0}\right|_{EF} = \frac{rw_0(u'(c_0^*) - u'(rA_0 - rk + w_0(1+e)))}{(rA_0 - c_0^*)u'(c_0^*) + w_0(1+e)u'(rA_0 - rk + w_0(1+e))} \geq 0$$

The inequality follows as both the numerator and the denominator are non-negative. As without education the workers save for retirement, (25) implies that $c_0^* \leq rA^0 - rk + w_0(1+e)$. Workers are willing to trade their prospect for future leisure only for a sufficiently large increase in consumption. As they work forever with education, $c_1^* \geq rA^0 - rk + w_0(1+e)$. Then for the denominator

$$\begin{aligned}(rA_0 - c_0^*)u'(c_0^*) + w_0(1+e)u'(rA_0 - rk + w_0(1+e)) \quad &\geq \\ (rA_0 - c_0^*)u'(c_0^*) + w_0(1+e)u'(c_1^*) \quad &= \\ (rA_0 + w_0 - c_0^*)u'(c_0^*) \quad &\geq \quad 0\end{aligned}$$

In the limit as $w_0 = rk/e$ and rA_0 approaches $c_0^*(rk/e) - rk/e$, (25) holds with equality and $\partial w_0/\partial A_0|_{EF}$ approaches zero. The education efficiency frontier for S_0^{FW} follows continuously from the frontier of FW_0 and describes an upward sloping curve in the $\{A_0, w_0\}$-plane.

Savers enjoy the benefit of boosted income only in annuity until retirement. Time to retirement declines with wealth and the wealthier a worker is, the larger is her present value of future leisure. A wealthier worker requires a larger increase in earnings to obtain education. As wealth increases workers indifferent to education have higher and higher ability.

B.2.2 Savers with education

The complement of S_0^{FW} in S_0

$$S_0^S \equiv \{A_0, w_0 | rA_0 \in (\max\{c_0^* - w_0, c_1 - w_0(1+e) + rk\}, \min\{c_0^*, c_1^* + rk\})\}$$

identifies workers whose optimal strategy is to save for retirement irrespective of education. The set is non-empty as long as $c^*(w_0) > c^*(w_0(1+e)) - w_0(1+e) + rk$ for some w_0.

Workers are indifferent to education if and only if

$$\left(\frac{(rA_0 + w_0 - c_0^*)}{w_0} - \frac{(rA_0 - rk + w_0(1+e) - c_1^*)}{(1+e)w_0} \right) u_b = u(c_1^*) - u(c_0^*) \tag{26}$$

that is, if the gain in consumption (in utility terms) just equals the loss of discounted value of leisure due to increase of time to retirement. Total differentiation with respect to A_0 implies

$$\left.\frac{\partial w_0}{\partial A_0}\right|_{EF} = \frac{rw_0(u'(c_0^*) - u'(c_1^*))}{u(c_1^*) - u(c_0^*)} > 0$$

The education efficiency frontier follows continuously from the one over S_0^{FW}.

B.3 Retirees without education

Consider the set of workers whose optimal strategy would be to retire immediately if they chose to obtain no education. They are identified by the set

$$R_0 \equiv \{A_0, w_0 | rA_0 \geq c_0^*\}$$

Should they invest in education instead their optimal strategy might be to work forever, save for retirement or still retire immediately.

The subset

$$R_0^R \equiv \{A_0, w_0 | rA_0 \geq c_1^* + rk\}$$

identifies the workers who retire immediately with or without education. As $u(rA_0) > u(rA_0 - rk), \forall A_0$ they never pursue a degree. If a worker retires immediately, the benefit of boosted earnings is not realised at all.

The subset

$$R_0^{FW} \equiv \{A_0, w_0 | rA_0 \in [c_0^*, c_1^* + rk - w_0(1+e))\}$$

identifies the workers who optimally work forever if they chose education. This set is non-empty.

Workers are indifferent to education if and only if

$$u(rA_0) + u_b = u(rA_0 - rk + w_0(1+e)) \tag{27}$$

Totally differentiating with respect to A_0

$$\left.\frac{\partial w_0}{\partial A_0}\right|_{EF} = \frac{r[u'(rA_0) - u'(rA_0 - rk + w_0(1+e))]}{(1+e)u'(rA_0 - rk + w_0(1+e))} > 0$$

The inequality follows because over (27) a forever-worker consumes more than a retiree for indifference to obtain.

The subset

$$R_0^{S} \equiv \{A_0, w_0 | rA_0 \in [\max\{c_0^*, c_1^* - w_0(1+e) + rk\}, c_1^* + rk\}$$

identifies workers who save for retirement if they chose education. This set is non-empty.

Workers are indifferent to education if and only if

$$u(rA_0) + u_b = u(c_1^*) + (rA_0 - rk + w_0(1+e) - c_1^*)u'(c_1^*) \tag{28}$$

Totally differentiating with respect to A_0 and using (28)

$$\left.\frac{\partial w_0}{\partial A_0}\right|_{EF} = \frac{rw_0[u'(rA_0) - u'(c_1^*)]}{u(c_1^*) - u'(rA_0)} > 0$$

The inequality follows as over (28) a worker will delay enjoyment of leisure only if they are able to attain higher consumption.

This completes the construction of the education efficiency frontier.

C Tables

Table 6: First stage

	1996		2001	
	Earnings$_t$	Saving$_t$	Earnings$_t$	Saving$_t$
Earnings$_{t-2}$			0.755***	0.158**
			(0.018)	(0.050)
Earnings$_{t-3}$	0.787***	0.098*		
	(0.026)	(0.043)		
Education				
HS,HS	1866.607	−3724.072	5330.778***	−720.917
	(1175.814)	(2228.318)	(974.846)	(1520.350)
<D,HS	1753.077	−2423.443	6279.136***	4571.777
	(1297.204)	(2265.825)	(1122.324)	(3051.242)
D+,HS	5094.001***	−8191.107*	10704.381***	1116.796
	(1513.423)	(4175.289)	(1804.790)	(5171.671)
HS,<D	2434.049	−2777.476	7644.515***	3352.449
	(1386.715)	(2797.656)	(1251.744)	(3016.435)
<D,<D	3700.294**	−2735.786	8009.959***	948.949
	(1295.635)	(2137.584)	(1096.206)	(2618.186)
D+,<D	8031.511***	3276.548	10872.854***	3780.029
	(1984.679)	(3281.118)	(1396.889)	(4381.975)
HS,D+	8414.996***	3052.624	11169.124***	3631.515
	(2133.960)	(5261.445)	(2941.858)	(6147.794)
<D,D+	6793.264***	−1380.225	9241.357***	−3193.662
	(1655.063)	(2856.892)	(1380.629)	(4078.870)
D+,D+	11651.517***	5503.179	14592.591***	8579.603*
	(1519.232)	(3160.614)	(1288.134)	(3443.893)
Age				
Age	1598.386***	−168.115	1724.869***	42.241
	(475.047)	(1016.471)	(470.307)	(1081.641)
Age^2	−18.508***	2.138	−19.290***	−1.266
	(5.490)	(12.169)	(5.349)	(12.823)
Net worth$_{t-1}$			0.005	0.027
			(0.003)	(0.018)
Net worth$_{t-2}$	0.003	0.021		
	(0.003)	(0.017)		
Intercept	−22299.932*	5553.979	−31880.985**	−2442.103
	(9893.479)	(20919.004)	(9959.012)	(22287.167)
Num. obs.	4946	4946	4780	4780

Bootstrap standard errors. Education categories: <HS - no high school, HS - high school, <D - higher education but no degree, D+ - at least a degree. First term in education interaction is household head's education category. Base category is <HS,<HS. Education categories that are insignificant in both equations are omitted.

Table 7: Second stage, interactions, DSLAD

	1996, p=0.2				2001, p=0.2			
	NW1	NW2	NW3	NW4	NW1	NW2	NW3	NW4
Earnings								
[0 − 25]	7961	7662	8900	10308	1978	1967	3405	7401
	(5522)	(5508)	(5499)	(5617)	(6007)	(5965)	(6018)	(6135)
(25 − 50]	10202	10662	11159*	15559**	6356	8051	9032	16369**
	(5474)	(5519)	(5586)	(5764)	(6061)	(6099)	(6108)	(6221)
(50 − 75]	13514*	14796**	16294**	21258***	11387	13359*	15697*	22347***
	(5585)	(5515)	(5568)	(5397)	(6095)	(6050)	(6145)	(6089)
(75 − 100]	18384***	23037***	27166***	39089***	19112**	24732***	29718***	43027***
	(5479)	(5540)	(5550)	(6137)	(6210)	(6289)	(6432)	(6788)
Age	-242				87			
	(262)				(286)			
Age^2	2				-2			
	(3)				(3)			
Num. obs.	4946				4780			

Bootstrap standard errors in parentheses. Controls omitted.